MACHAKOS UNIVERSITY

University Examinations 2021/2022 Academic Year

SCHOOL OF PURE AND APPLIED SCIENCES

DEPARTMENT OF MATHEMATICS AND STATISTICS

THIRD YEAR FIRST SEMESTER EXAMINATION FOR

BACHELOR OF SCIENCE (MATHEMATICS AND COMPUTER SCIENCE)

BACHELOR OF EDUCATION (SPECIAL NEEDS EDUCATION)

BACHELOR OF SCIENCE (MATHEMATICS)

BACHELOR OF EDUCATION (SCIENCE)

BACHELOR OF EDUCATION (ARTS)

SMA 333: FLUID MECHANICS I

DATE: 24/8/2022 **TIME: 2.00-4.00 PM**

INSTRUCTION:

Answer Question One and Any Other Two Questions

QUESTION ONE (30 MARKS)

a) A plate at a distance of $0.2\ cm$ from a fixed point. The plate moves at and requires a force $40\, dyres/cm^2$ of to maintain that speed. Determine the coefficient of viscosity of the fluid between the plates. (4 marks)

b) Determine the equation of streamline passing through the point $(a, 0)$ if the 2-D flow is described by; $u = \frac{-y}{b^2}$, $v = \frac{x}{a^2}$ (4 marks)

c) Given the velocity fields;

$$V_r = 4\cos\theta\left(1 - \frac{1}{r^2}\right), \qquad V_\theta = -4\sin\theta\left(1 + \frac{1}{r^2}\right), \qquad V_z = 0$$

Show that the velocity field represent a possible incompressible flow (5 marks)

d) Prove that $C_p - C_v = \rho$ (5 marks)

e) Calculate the velocity field of a spherically symmetric flow whose stream function is $\psi(r,\theta)=\frac{Ua^3}{2r}\sin^3\theta-\frac{Ur^2}{2}\sin^2\theta$ (5 marks)

f) The x component of the velocity in a certain plane flow depends on y by the relationship $u(y)=Ay$. Determine the y component $v(x,\ y)$ of the velocity if $v(x,0)=0$ (5 marks)

g) Determine whether the fluid with vorticity component $u=\frac{x}{x^2+y^2}$, $v=\frac{x}{x^2+y^2}$, is irrotational. (5 marks)

QUESTION TWO (20 MARKS)

a) Determine the analytical function $f(z)=u+iv$, given that;

$u=x^3-3y^2+3x^2-3y^2+2x+1$ (6 marks)

b) Given that $w=u+iv$ is the fluids complex potential function, and that $f(z)$ is an analytic complex function with $z=x+i\,y$. Derive the;

i. The Cauchy -Riemann's set of equations for the complex velocity potential function (7 marks)

ii. Laplace's equation for the complex velocity potential function (7 marks)

QUESTION THREE (20 MARKS)

The ideal gas obeying Boyle's law $P=k\rho$ is rushing from a boiler through a conical pipe with radii $a\ and\ b$; $(b>a)$ at the two ends. If the gas enters a pipe from the narrow end with the velocity V and escapes through the other end with velocity U, show that $V=\frac{Ub^2}{a^2}\exp\left[\frac{V^2-U^2}{2k}\right]$

QUESTION FOUR (20 MARKS)

A two-dimensional flow towards a normal boundary is found to be characterized by a normal component of the velocity that varies directly with distance from the boundary. Determine;

a) The stream function (11 marks)

b) The stream line (9 marks)

QUESTION FIVE (20 MARKS)

Given the velocity of an incompressible fluid at the point $(x,\ y,\ z)$ as;

$$u = \frac{3xz}{r^5}, \quad v = \frac{3yz}{r^5}, \quad w = \frac{3z^2 - r^2}{r^5}$$

Show that;

a) The fluid motion is possible (12 marks)

b) The velocity potential is $\frac{-\cos\theta}{r^2}$; $if \quad z = r\cos\theta$ (8 marks)