MACHAKOS UNIVERSITY

University Examinations 2021/2022 Academic Year

SCHOOL OF PURE AND APPLIED SCIENCES

DEPARTMENT OF MATHEMATICS AND STATISTICS

FOURTH YEAR FIRST SEMESTER EXAMINATION FOR

BACHELOR OF SCIENCE (MATHEMATICS AND COMPUTER SCIENCE)

BACHELOR OF EDUCATION (SPECIAL NEEDS EDUCATION)

BACHELOR OF SCIENCE (MATHEMATICS)

BACHELOR OF EDUCATION (SCIENCE)

BACHELOR OF EDUCATION (ARTS)

BACHELOR OF ARTS

SMA 431: DIFFERENTIAL GEOMETRY

DATE: 26/8/2022 **TIME: 11.00-1.00 PM**

INSTRUCTION:

Answer Question One and Any Other Two Questions

QUESTION ONE (30 MARKS)

a) The volume of a parallelepiped is 4 unit. If its edges are;

$\vec{A} = 2\hat{i} - 3\hat{j} - 2\hat{k}$; $\vec{B} = 2\hat{i} + 3\hat{j} + 3\hat{k}$; $\vec{C} = 4\hat{i} - \beta\hat{j} + 2\beta\hat{k}$.

Determine the value of β (4 marks)

b) Given four vectors $\vec{A}$, $\vec{B}$, $\vec{C}$ *and* $\vec{D}$. Show that

$\left(\vec{B} \times \vec{C}\right) \cdot \left(\vec{A} \times \vec{D}\right) + \left(\vec{C} \times \vec{A}\right) \cdot \left(\vec{B} \times \vec{D}\right) + \left(\vec{A} \times \vec{B}\right) \cdot \left(\vec{C} \times \vec{D}\right) = 0$ (4 marks)

c) Given that the vector $\vec{A} = (a, b, c)$ have direction cosines $\cos\alpha$, $\cos\beta$, $\cos\gamma$. Evaluate;

$\cos^2\alpha + \cos^2\beta + \cos^2\gamma$ (4 marks)

d) Consider the curve $x = 3t$, $y = 3t^2$, $z = 2t^3$ at the point $t = 1$. Determine;

i. The unit tangent vector to the curve $\hat{T}$ (4 marks)

ii. The binormal vector to the curve $\hat{B}$ (4 marks)

e) Consider the surface described by the equation $R = e^t \cos t\,\hat{i} + e^t \sin t\,\hat{j} + e^t\,\hat{k}$ from $0 \le t \le \pi$. Determine the arc length of the surface in the given interval. (5 marks)

f) Determine the second fundamental form on the surface $R = u\,\hat{i} + v\,\hat{j} + (u^2 - v^2)\,\hat{k}$ (5 marks)

QUESTION TWO (20 MARKS)

a) Consider the line through the point $(1,\ 2,\ -6)$ and parallel to the vector $(4,\ 1,\ 3)$. Work out the;

   i. Line's vector equation (3 marks)

   ii. Line's parametric equation (4 marks)

   iii. Line's rectangular equation (4 marks)

b) Consider the plane through the points $A\ (3, 2, -1),\ \ B(1, -1, 3),\ \ C(3, -2,\ 4)$. Calculate the plane's equation (9 marks)

QUESTION THREE (20 MARKS)

Given the space curve equation $R = 3\cos t\,\hat{i} + 3\sin t\,\hat{j} + 4t\,\hat{k}$. Determine;

a) The principal normal vector $\hat{N}$ (5 marks)

b) The curvature κ (5 marks)

c) The torsion τ (5 marks)

d) The radius of torsion σ (5 marks)

QUESTION FOUR (20 MARKS)

Consider the surface described by the function $R = u\,\hat{i} + 2v\,\hat{j} + 2uv\,\hat{k}$. Determine the surface's;

a) first fundamental magnitudes (10 marks)

b) second fundamental magnitudes (10 marks)

QUESTION FIVE (20 MARKS)

Given the space curve $x = 2t^2\hat{i} + 3t\,\hat{j} + 3t^2\hat{k}$. If the curve passes $(2,\ 3,\ 3)$. Calculate the curves;

a) normal plane equation (5 marks)

b) osculating plane equation (7 marks)

c) rectifying plane equation (8 marks)