MACHAKOS UNIVERSITY

University Examinations 2021/2022 Academic Year

SCHOOL OF PURE AND APPLIED SCIENCES

DEPARTMENT OF MATHEMATICS AND STATISTICS

THIRD YEAR FIRST SEMESTER EXAMINATION FOR

BACHELOR OF SCIENCE (MATHEMATICS AND COMPUTER SCIENCE)

BACHELOR OF SCIENCE (STATISTICS AND PROGRAMMING)

BACHELOR OF EDUCATION (SPECIAL NEED EDUCATION)

BACHELOR OF SCIENCE (MATHEMATICS)

BACHELOR OF EDUCATION (SCIENCE)

BACHELOR OF EDUCATION (ARTS)

SMA335: ORDINARY DIFFERENTIAL EQUATIONS I

DATE: 25/8/2022 **TIME: 8.30-10.30 AM**

INSTRUCTION:

Answer Question One and Any Other Two Questions

QUESTION ONE (30 MARKS)

a) Determine the differential equation associated with the primitive $y = cx^2 + \sin x$ (4 marks)

b) Solve the differential equation $\frac{dy}{dx} + \frac{x}{y} = 0$ (4 marks)

c) Determine the orthogonal trajectories of the family of curves $x^2 + y^2 = c^2$. c is an arbitrary constant. (4 marks)

d) Determine a particular solution to the second order differential equation
$(D^2 - 8D + 16)y = 0$ (4 marks)

e) Calculate the integrating factor for the equation
$\frac{dq}{dt} + \frac{2}{10+2t}q = 4$ (4 marks)

f) Transform the equation below to its homogeneous form

$$\frac{dy}{dx} = \frac{x - y + 3}{2x - 2y + 5}$$ (5 marks)

g) An RC circuit has an $e.m.f$ given (in volts) by $400\cos 2t$, resistance of 100 $Ohms$, and a capacitance of 10^{-2} *faradays.* Initially there is no charge on the capacitor. Determine the current in the circuit at any time t (5 marks)

QUESTION TWO (20 MARKS)

a) Define 1^{st} order exact differential equation (2 marks)

b) Prove the necessary and sufficient condition for exactness theorem;

$$\frac{\partial M(x,y)}{\partial y} = \frac{\partial N(x,y)}{\partial x}$$ (7 marks)

c) Solve the equation $(1 + e^{\frac{x}{y}})dx + e^{\frac{x}{y}}(1 - \frac{x}{y})dy = 0$ (11 marks)

QUESTION THREE (20 MARKS)

A metal bar at temperature of $50^0 F$ is placed outdoors where the temperature is $100^0 F$. If after 50 minutes the temperature of the body is $60^0 F$. Determine;

a) The time it takes the body to attain a temperature of $75^0 F$ (14 marks)

b) The temperature of the body after $20\,\text{min}$ (6 marks)

QUESTION FOUR (20 MARKS)

Solve the differential equations with variable coefficients below

a) $(x+a)^2 \frac{dy}{dx} - 4(x+a)\frac{dy}{dx} + 6y = x$ (8 marks)

b) $x^2 \frac{d^2y}{dx^2} + 7x\frac{dy}{dx} + 13y = e^{\log x}$ (12 marks)

QUESTION FIVE (20 MARKS)

a) Define the n^{th} *order* linear differential equation with constant coefficient (3 marks)

b) $\frac{d^4y}{dx^4} + 2\frac{d^3y}{dx^3} + \frac{d^2y}{dx^2} = \sin 3x$ (7 marks)

c) $(D^2 - 4D + 4)y = 8x^2 e^{2x} \sin 2x$ (10 marks)

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