

DATE: 25/8/2022

TIME: 8.30-10.30 AM

## **INSTRUCTION:**

## Answer Question One and Any Other Two Questions

## **QUESTION ONE (30 MARKS)**

a)	Determine the differential equation associated with the primitive $y = cx^2 + \sin x$	(4 marks)	
b)	Solve the differential equation $\frac{dy}{dx} + \frac{x}{y} = 0$	(4 marks)	
c)	Determine the orthogonal trajectories of the family of curves $x^2 + y^2 = c^2$ . <i>c</i> is an arbitrary		
	constant.	(4 marks)	
d)	Determine a particular solution to the second order differential equation		
	$(D^2 - 8D + 16)y = 0$	(4 marks)	
e)	Calculate the integrating factor for the equation		
	$\frac{dq}{dt} + \frac{2}{10+2t}q = 4$	(4 marks)	

f) Transform the equation below to its homogeneous form

$$\frac{dy}{dx} = \frac{x - y + 3}{2x - 2y + 5} \tag{5 marks}$$

g) An *RC* circuit has an *e.m.f* given (in volts) by  $400 \cos 2t$ , resistance of 100 Ohms, and a capacitance of  $10^{-2}$  *faradays*. Initially there is no charge on the capacitor. Determine the current in the circuit at any time *t* (5 marks)

# **QUESTION TWO (20 MARKS)**

a) Define 
$$1^{st}$$
 order exact differential equation (2 marks)

b) Prove the necessary and sufficient condition for exactness theorem;

$$\frac{\partial M(x, y)}{\partial y} = \frac{\partial N(x, y)}{\partial x}$$
(7 marks)

c) Solve the equation 
$$(1 + e^{\frac{1}{y}})dx + e^{\frac{1}{y}}(1 - \frac{x}{y})dy = 0$$
 (11 marks)

#### **QUESTION THREE (20 MARKS)**

A metal bar at temperature of  $50^{\circ}F$  is placed outdoors where the temperature is  $100^{\circ}F$ . If after 50 minutes the temperature of the body is  $60^{\circ}F$ . Determine;

- a) The time it takes the body to attain a temperature of  $75^{\circ}F$  (14 marks)
- b) The temperature of the body after 20 min (6 marks)

#### **QUESTION FOUR (20 MARKS)**

Solve the differential equations with variable coefficients below

a) 
$$(x+a)^2 \frac{dy}{dx} - 4(x+a)\frac{dy}{dx} + 6y = x$$
 (8 marks)

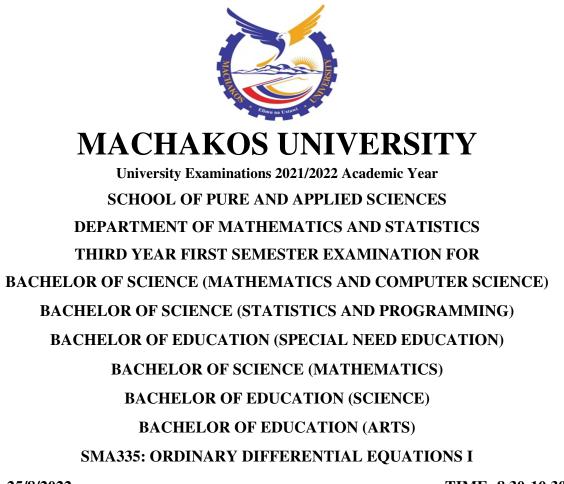
b) 
$$x^2 \frac{d^2 y}{dx^2} + 7x \frac{dy}{dx} + 13y = e^{\log x}$$
 (12 marks)

## **QUESTION FIVE (20 MARKS)**

a)	Define the $n^{th}$	order linear differential equation with constant coefficient	(3 marks)

b) 
$$\frac{d^4y}{dx^4} + 2\frac{d^3y}{dx^3} + \frac{d^2y}{dx^2} = \sin 3x$$
 (7 marks)

c) 
$$(D^2 - 4D + 4)y = 8x^2 e^{2x} \sin 2x$$
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