MACHAKOS UNIVERSITY

University Examinations 2021/2022 Academic Year

SCHOOL OF PURE AND APPLIED SCIENCES

DEPARTMENT OF MATHEMATICS AND STATISTICS

FOURTH YEAR FIRST SEMESTER EXAMINATION FOR

BACHELOR OF SCIENCE (STATISTICS AND PROGRAMMING)

SMA 465: INTRODUCTION TO MEASURE AND PROBABILITY

DATE: 26/8/2022 **TIME: 2.00-4.00 PM**

INSTRUCTION:

Answer **ALL** Questions from **Section A** and any other **TWO** Questions in **Section B**.
All questions in **Section B** carry equal marks.
Start each Question on a **new** page.

SECTION A (30 Marks). Answer ALL questions.

QUESTION ONE (30 MARKS)

a) Define the following terms as used in measure theory and probability

i. A closed Set (1 mark)
ii. σ- algebra
iii. Event (1 mark)
iv. Measure (1 mark)
v. Sample space (1 mark)
vi. Interval additive property (1 mark)

b) State the fundamental theorem of Lebesque measure on $\mathbb{R}^n$ (3 marks)

c) List any two axioms of measure space (2 marks)

d) Show that If $A, B \epsilon P$ then $\mu(B) = \mu(A \cap B) + \mu(B \backslash A)$ (2 marks)

e) Describe briefly the fundamental theorems of integral functions in measure theory. (2 marks)

f) Let X_1, X_2, X_3 ... be a sequence of independently identical distributed random variables($i.i.i\ random\ variables$) each with the same distribution, each having common mean $\mu = (X)$ and variance $\delta^2 = Va(X)$. Show that $(X) = (X) \times E(N) = \mu E(N)$, where $S = \sum^N \ j{=}1\ X_j$, and the number in the sum, N is also a random variable and is independent of the

X_{j}, S. (4 marks)

g) Give any two application of measure (2 marks)

h) Describe any two types of measure with well-illustrated examples. (4 marks)

i) Differentiate between the following

i. measurable sets and measurable functions (2 marks)

ii. measurable space and measure space (2 marks)

iii. probability space and probability measure (2 marks)

SECTION B (40 Marks). Answer any TWO questions

QUESTION TWO (20 MARKS)

a) Define the conditional expectation of random variables (3 marks)

b) State any four properties of conditional expectation (2 marks)

c) State and proof the law of large numbers (4 marks)

d) Let X and Y be two independent random variables, show that E(X/Y) = E(Y) (3 marks)

e) Let X, Y Z be jointly distributed with pdf,

$$f(xyz) = \begin{cases} \frac{1}{(2\pi)^{3/2}} e^{-\frac{1}{2}(X^2\, Y^2\, Z^2)} & -\infty < x, y, z < \infty \\ 0 \ else\ where \end{cases}$$

Find f(X/YZ), f(Y/XZ) , and f(Z/XY) (9 marks)

QUESTION THREE (20 Marks)

a) What are Lebasque measurable sets (2 marks)

b) Describe any two examples of Lebasque measurable sets (4 marks)

c) State the central limit theorem and its importance in sampling distributions (2 marks)

d) State and prove the Fatou's Lemma (5 marks)

e) State and proof the law of large numbers (7 marks)

QUESTION FOUR (20 MARKS)

a) Define the characteristic function of a random variable (1 mark)

b) If X is normally distributed with mean μ and variance δ^2,

i. Obtain the characteristic function of X; (8 marks)

ii. Using the characteristic function, find the mean of X (2 marks)

iii. Hence find the variance of X (4 marks)

c) With well-illustrated examples show that the length, area and volume possess additive properties in measure theory. (5 marks)

QUESTION FIVE (20 MARKS)

a) What is an integral function (3 marks)

b) state any five properties of inferable functions (5 marks)

c) describe the two modes of convergence theorems (6 marks)

d) Define μ – finite measure (3 marks)

e) Describe Lebasque measure (3 marks)