# MACHAKOS UNIVERSITY

**University Examinations 2021/2022 Academic Year**

**SCHOOL OF PURE AND APPLIED SCIENCES**

**DEPARTMENT OF MATHEMATICS AND STATISTICS**

**FOURTH YEAR FIRST SEMESTER EXAMINATION FOR**

**BACHELOR OF SCIENCE (MATHEMATICS AND COMPUTER SCIENCE)**

**BACHELOR OF SCIENCE (MATHEMATICS)**

**BACHELOR OF EDUCATION**

**SMA 400: TOPOLOGY I**

**DATE: 31/8/2022** **TIME: 2.00-4.00 PM**

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**INSTRUCTION:**
***Answer Question One and Any Other Two Questions***

**QUESTION ONE (30 MARKS) COMPULSORY**

a) Let $\tau = \{X, \emptyset, \{a\}, \{a,b\}, \{a,c,d\}, \{a,b,c,d\}, \{a,b,e\}\}$be a topology on $X = \{a,b,c,d,e\}$. Determine the neighbourhood system of the point $e$ and c (4 marks)

b) Let $X = \{a, b, c, d, e\}$ and $\rho = \{\emptyset, X, \{a\}, \{b\}, \{a,b\}, \{c\}, \{abc\}, \{a,c\}, \{b,c\}\}$.

Let $E = \{a,b,c\}$, Determine:

i The closure of E (4 marks)

ii The interior of E (4 marks)

c) Let $\tau_1 = \{X, \emptyset, \{a\}, \{c,d\}, \{a,c,d\}, \{b,c,d,e\}\}$ be a topology on $X = \{a,b,c,d,e\}$ and Let $A = \{a,b,c\} \subset X$, determine $A'$, the derived set of $A$. (5 marks)

d) Let $X = \{1,2,3,4\}$ and $S = \{\{1\}, \{2,3\}, \{3,4\}\}$ be a collection of subsets of $X$. Give the topology generated by the set $S$. (5 marks)

e) Define the neighborhood of a point $p \in X$ where $X$ is a topological space. (2 marks)

f) Determine the topology generated in R by the class of all closed sub-intervals of the type $(x, x+1)$ (4 marks)

g) Let $X$ be a non-empty set. Define the following topologies

i. Trivial topology

ii. Discrete topology (2 marks)

**QUESTION TWO (20 MARKS)**

a) Consider the topology $\tau = \{X, \emptyset, \{1\}, \{3,4\}, \{1,3,4\}, \{1,3,4,5\}\}$ on $X = \{1,2,3,4,5\}$ and the subset

$A = \{2,4,5\}$ of $X$. Compute showing working out:

i. $Int(A)$ (3 marks)

ii. $Ext(A)$ (3 marks)

iii. $B'dary(A)$ (4 marks)

b) Let $(X, \tau)$ be a topological space. Define a base and sub-base for the topology τ on $X$. (2 marks)

c) Let $X = \{1,2,3\}$ and $\tau = \{\{1,2\}, \{2,3\}, \emptyset, \{2\}, \{1,2,3\}\}$. Give the base and sub-base for the topology $\tau$ on $X$. (4 marks)

d) Given a subset $A$ of a topological space, define what it means for $p$ to be a limit point of $A$. (2 marks)

e) Let $X$ be a topological space and $A \subseteq X$. Give the definition of the closure of $A$. (2 marks)

**QUESTION THREE (20 MARKS)**

a) Determine the topology generated by the sub-basis $\beta = \{\{1\}, \{1, 3, 4\}, \{2, 3\}, \{3\}\}$, where $X = \{1, 2, 3, 4\}$ (6 marks)

b) Given any collection of subsets of a nonvoid set X. Will this collection serve as a base for the topology? (6 marks)

c) If $N_1$ $and$ $N_2$ are two neighborhoods of x, show that $N_1 \cap N_2$ is also a neighborhood of x. (4 marks)

d) In a topological space $X$, a subset $A$ $of$ $X$ is open if only if its complement is closed. (4 marks)

## QUESTION FOUR (20 MARKS)

a) Let $X = \{a, b, c, d, e\}$ and $J = \{\emptyset, x\{a\}\{b, c\}\{b, c, d\}\{a, b, c\}\{a, d\}\{d\}\{a, b, c, d\}\}$ let $Y = \{a, b, c, d\}$,

i. Determine the induced topology on Y

ii. Define relative topology

iii. Determine the closed set in relative topology (6 marks)

b) Define hereditary as used in topological spaces (2 marks)

c) $T_{0\ and}\ T_1$ spaces are hereditary prove (4 marks)

d) Let $(X, \rho)$ be a topological space $(X, \rho)$ is a $T_1$ space iff each singleton subset $\{x\}$ is closed in $(X, \rho)$ (4 marks)

e) Consider the discrete topology $\tau$ on $X = \{a, b, c, d\}$. Find a sub-base for $\tau$ which does not contain any singleton sets. (4 marks)

## QUESTION FIVE (20 MARKS)

a) Let $(X, \partial)$ be a metric space and A $\subset X$. Then prove that if P is a limit point of A every neighborhood of P contains infinitely many points distinct from P. (8 marks)

b) Give the definition of a homeomorphism between two topological spaces $X$ and $Y$. (4 marks)

c) Let $(X, \rho)$ and $(Y, \rho^*)$ be topological spaces and $f: X \to Y$ be a function if $p \in X$ and $\{\ p\} \in \rho$ Show that f is continuous. (8 marks)