MACHAKOS UNIVERSITY

University Examinations 2021/2022 Academic Year

SCHOOL OF PURE AND APPLIED SCIENCES

DEPARTMENT OF MATHEMATICS AND STATISTICS

THIRD YEAR FIRST SEMESTER EXAMINATION FOR

BACHELOR OF SCIENCE (ELECTRICAL AND ELECTRONIC ENGINEERING)

BACHELOR OF SCIENCE (MECHANICAL ENGINEERING)

BACHELOR OF SCIENCE (CIVIL ENGINEERING)

ECU 300: ENGINEERING MATHEMATICS IX

DATE: 29/8/2022 **TIME: 8.30-10.30 AM**

INSTRUCTION:

Answer Question One and Any Other Two Questions

QUESTION ONE (30 MARKS)

a) Show that $\vec{a} \times \vec{b}$ is orthogonal to $\vec{a}$ and $\vec{b}$ (5 marks)

b) Prove that $Grad\ \vec{r} = \frac{\vec{r}}{r}$ where $\vec{r} = x\hat{\imath} + y\hat{\jmath} + z\hat{k}$ and $r = (x^2 + y^2 + z^2)^{\frac{1}{2}}$ (5 marks)

c) If $\vec{V} = v_1\hat{\imath} + v_2\hat{\jmath} + v_3\hat{k}$, show that $\nabla \cdot \left(\nabla \times \vec{V}\right) = 0$ (4 marks)

d) Evaluate the scalar and vector projections of $\vec{b}$ onto $\vec{a}$ if $\vec{a} = (-2,3,-6)$ and $\vec{b} = (5,-1,4)$ (4 marks)

e) Find the directional derivative of $\varphi = (x + 2y + z)^2 - (x - y - z)^2$ at the point (2,1,-1) in the direction of $\boldsymbol{A} = \boldsymbol{i} - 4\boldsymbol{j} + 2\boldsymbol{k}$. (4 marks)

f) Verify Divergence theorem for $\vec{F} = x^2\boldsymbol{i} + y^2\boldsymbol{j} + z^2\boldsymbol{k}$ taken on the unit cube $0 \leq x, y, z \leq 1$ (5 marks)

g) Convert $\left(8, \frac{\pi}{4}, \frac{\pi}{6}\right)$ from spherical to Cartesian coordinates (3 marks)

QUESTION TWO (20 MARKS)

a) Show that $\lim_{t\to 1} \vec{r}(t) = \frac{t^2-t}{t-1}\hat{\imath} + \sqrt{t+8}\,\hat{\jmath} + \frac{Sin\,\pi t}{ln\,t}\hat{k}$ exists but $\vec{r}(t)$ is not continuous at $t = 1$

(5 marks)

b) Find $\frac{dz}{dt}$ if $z = ln(3x^2 + y^3), x = e^{2t}, y = t^{\frac{1}{3}}$ (7 marks)

c) Find $\frac{\partial z}{\partial u}, \frac{\partial z}{\partial v}$ if $z = e^{xy}, x(u,v) = 3u\,sin\,v,\ y(u,v) = 4v^2u$ (8 marks)

QUESTION THREE (20 MARKS)

a) Find and classify the stationary points of $f(x,y) = x^2y + 3xy^2 - 3xy$ (10 marks)

b) Determine the volume of the solid that lies under the surface $z = xy$ and above the triangle with vertices $(1,1)$, $(4,1)$ and $(1,2)$. (10 marks)

QUESTION FOUR (20 MARKS)

a) Determine whether the vector field $\vec{F}(x,y) = (3x^2y - \cos x + 4, x^3 - e^y)$ is conservative and if so find its scalar potential function (10 marks)

b) Verify Green's theorem in the plane for $\oint_c\ (xy + y^2)dx + x^2\,dy$ where c is the closed curve of the region bounded by $y = x$ and $y = x^2$. (10 marks)

QUESTION FIVE (20 MARKS)

a) Show that the scale factors in cylindrical coordinate system are given by $h_r = 1$, $h_\theta = r$ and $h_z = 1$. (6 marks)

b) Represent $\vec{A} = z\boldsymbol{i} - 2x\boldsymbol{j} + y\boldsymbol{k}$ in cylindrical coordinates and determine A_r, A_θ and A_z. (7 marks)

c) Evaluate $\iiint ydV$ over region E which is below the plane $z = x + 2$ and above the $xy -$ plane and between the cylinders $x^2 + y^2 = 1$ and $x^2 + y^2 = 4$. (7 marks)