MACHAKOS UNIVERSITY

University Examinations 2021/2022 Academic Year

SCHOOL OF PURE AND APPLIED SCIENCES

DEPARTMENT OF MATHEMATICS AND STATISTICS

THIRD YEAR FIRST SEMESTER EXAMINATION FOR

BACHELOR OF SCIENCE (MATHEMATICS AND COMPUTER SCIENCE)

BACHELOR OF SCIENCE (STATISTICS AND PROGRAMMING)

BACHELOR OF SCIENCE (ACTUARIAL SCIENCE)

BACHELOR OF SCIENCE (MATHEMATICS)

BACHELOR OF EDUCATION

BACHELOR OF ARTS

SMA 361-THEORY OF ESTIMATION

DATE: 23/7/2022 **TIME: 8.30-10.30 AM**

INSTRUCTION:

Answer Question One and Any Other Two Questions

QUESTION ONE (COMPULSORY)

a) $Let\ x_1, x_2, .. x_n$be identically independent random variable with a finite mean μ,show that the sample mean $\bar{x}$ is always unbiased estimator of μ. (4 marks)

b) Let $x_1, x_2, .. x_n$ denote a random sample from a pdf is $f(x, \theta) = \theta x^{\theta-1}$,0$< x < 1, \theta > 0$

Show that $t_1 = \prod_{i-1}^{n} x_i$ is sufficient for θ (4 marks)

c) $Let\ x_1, x_2, .. x_n$ be identically independent random variable with a finite variance δ^2 .Show that the sample variance $s^2 = \frac{1}{n-1}(\sum_1^n (x - \bar{x})^2$=$\frac{1}{n-1}\{\sum_1^n x^2 - n\bar{x}^2\}$ (4 marks)

d) $Let\ x_1, x_2, .. x_n$be a random sample drawn from a population with mean μ (unknown).Show that if $y = \sum_1^n a_i x_i$ is unbiased estimator for μ if $\sum_1^n a_i = 1$ (4 marks)

e) $Let\ x_1, x_2, ..x_n$ be a random sample from a population with mean μ and variance δ^2. Consider the following estimators forμ.

$$t_1 = {}^1/_2\,(x_1 + x_2)$$

$$t_2 = \frac{\frac{1}{2}x_1 + x_2 + \cdots + x_{n-1}}{2(n-1)}$$

$$t_3 = \bar{x}$$

- Show that each of 3 estimators is unbiased. (5 marks)
- Determine the efficiency of t_3 relative to t_2 (5 marks)

f) Let $x_1, x_2, ..x_n$ be a random sample of size n from a distribution with mean from a population with mean μ (unknown) and known variance δ^2.Determine the maximum likelihood estimator (MLE) of μ (4 marks)

QUESTION TWO (20 MARKS)

Three objects M, N and P with weights$t_{1,} t_2\ and,t_3$ respectively are weighed on a spring balance M and N together weigh 75g, M and P weigh 45g and N and P weigh 100g. Assuming the weights are independent with constant variance δ^2, determine the least square estimates of $t_1, t_2, and\ t_3$

QUESTION THREE (20 MARKS)

a) Let $x_1, x_2, ..x_n$ be a random sample drawn from a population with an exponential probability density function given by $f(x) = \lambda e^{-\lambda x}, x > 0$ .show that $Y = \sum_{I=1}^{n} X_I$ is sufficient for λ (8 marks)

b) A random sample of size n is drawn from a normal population with (0,δ^2). Determine the minimum variance unbiased estimator (MVUE) of δ^2 and its variance. (12 marks)

QUESTION FOUR (20 MARKS)

a) Let $x_1, x_2, ..x_n$ denote a random sample from a pdf

$$f(x) = \begin{bmatrix} (\lambda+1)x^{\lambda}, & 0 < \text{x} < 1,\ \lambda > 0 \\ 0 & elsewhere \end{bmatrix}$$

Obtain the moment estimator of λ and test for unbiasedness (10 marks)

b) Let X be random variable that is binomially distributed with a pdf defined as

$$f(x;\theta) = \binom{n}{x}\theta^x(1-\theta)^{n-x} \quad for\ x = 0,1 \ldots n, \theta \in (0,1).$$

Determine the Crammer-Rao lower bound (10 marks)

QUESTION FIVE (20 MARKS)

a) A population has a known mean μ and a standard deviation δ of 2.45. If a sample of 900 has a mean of 3.15cm and the population is normal obtain a 98% confidence intervals for the true mean. (10 marks)

b) If $x_1, x_2, .. x_n$ is a random sample of size n from a population whose pdf is given by

$$f(x, \lambda) = \frac{e^{-\lambda}\lambda^x}{x!}, x = 0, 1, 2, ...$$

Determine the uniformly minimum variance unbiased estimator (UMVUE) of $e^{-\lambda}$(10 marks)