MACHAKOS UNIVERSITY

University Examinations 2021/2022 Academic Year

SCHOOL OF PURE AND APPLIED SCIENCES

DEPARTMENT OF MATHEMATICS AND STATISTICS

THIRD YEAR FIRST SEMESTER EXAMINATION FOR

BACHELOR OF SCIENCE (ACTUARIAL SCIENCE)

SMA 362: OPERATIONS RESEARCH I

DATE: 1/9/2022 **TIME: 8.30-10.30 AM**

INSTRUCTION:

Answer Question One and Any Other Two Questions

You must have a Scientific Calculator and Graph Paper.

QUESTION ONE (30 MARKS)

(a) Outline **four** areas of application of Operations Research techniques as tools for management function. (4 *marks*)

(b) (i) Differentiate between a *slack variable* and a *surplus variable* as used in linear programming. (4 *marks*)

(ii) Explain the implications of having an artificial variable with a non-zero value in an optimal solution to a linear programming problem solved using the Simplex method. (2 *marks*)

(c) Given the linear programming model below:

Minimise : $z = 12x_1 + 8x_2 + 10x_3$

Subject to : $5x_1 + 2x_2 + 6x_3 \geq 40$

$3x_1 + 4x_2 + 2x_3 \geq 25$

$4x_1 + 5x_2 + 3x_3 \geq 50$

With : $x_1, \quad x_2, \quad x_3 \geq 0$

Determine its symmetrical dual program. (2 *marks*)

(d) A soft drink production plant produces two quantities of soda: 300 ml and 500 ml. The sale prices per bottle are Ksh 30 for 300 ml and Ksh 45 for 500 ml. The firm wants to determine the hourly production plan which maximises contribution to its sales revenue. The production constraints are as shown below:

	Machine hours	Labour hours	Colouring agent
300 ml	1	2	1
500 ml	2	1	1
Maximum available	120	140	80

A customer has placed an order that a minimum of 20 bottles of 300 ml must be produced per hour in conformity with their consumption.

(i) Formulate a linear programming model for the problem. (3 *marks*)

(ii) Using the graphical method, determine the optimum production plan which maximises the hourly revenue for the bottling firm. (6 *marks*)

(iii) Interpret the optimum solution obtained in (ii) above. (3 *marks*)

(e) A shipping company is required to meet the demands at various destinations by transporting 80, 100, 40, 120, 60 containers respectively from various supply sources with 120, 80, 160, 40 containers. The transport cost in thousand Kenya shillings per unit of container over the various routes are as given in the matrix below:

$$C = \begin{bmatrix} 4 & 8 & 7 & 1 & 6 \\ 2 & 6 & 5 & 4 & 2 \\ 6 & 2 & 8 & 3 & 5 \\ 5 & 1 & 2 & 7 & 3 \end{bmatrix}$$

The company wants to meet the demand at the destinations by transporting the containers at the cheapest cost possible. Derive the *initial basic solution* and its corresponding total cost of transport using each of the following methods:

(i) Northwest corner (NWC) rule;

(ii) Vogel's approximation method (VAM). (6 *marks*)

QUESTION TWO (20 MARKS)

A fruit farm which produces three types of fruit juices – mango, orange and pineapple, has its daily production plan for sales revenue modelled as an LP program as shown below.:

Maximise : $z = 80x_1 + 120x_2 + 150x_3$

Subject to :	$2x_1 + 3x_2 + x_3 \leq 1500$	Machine hours
	$2x_1 + x_2 + 5x_3 \leq 1200$	Preservative
	$x_1 + 2x_2 + x_3 \leq 750$	Colouring agent
	$x_2 \leq 250$	Trade agreement
With :	$x_1, \ x_2, \ x_3 \geq 0$	Non-negativity conditions

where x_1 represents the number of packets of mango juice produced,

x_2 represents the number of packets of orange juice produced, and

x_3 represents the number of packets of pineapple juice produced.

(i) Using the Simplex method, determine the optimum daily production plan for the farm. (16 *marks*)

(ii) Interpret the optimum solution obtained in (i) above. (4 *marks*)

QUESTION THREE (20 MARKS)

Below is a linear programming model for a given problem. Use it to answer the questions that follow.

Minimise : $z = 180x_1 + 120x_2 + 80x_3$

Subject to : $2x_1 + 2x_2 + x_3 \geq 80$

$3x_1 + x_2 + x_3 \geq 70$

With : $x_1, \ x_2, \ x_3 \geq 0$

a) Explain the rationale for the use of *duality* in solving a linear programming problem using the simplex technique. (2 *marks*)

b) Determine the *symmetrical dual* of the linear programming model above. (2 *marks*)

c) Using the Simplex method, determine the optimum solution of the dual program for the linear programming model above. (9 *marks*)

d) Using the relationship between a primal program and its symmetrical dual program, extract the optimum solution of the primal program from the optimum solution of its dual program. (4 *marks*)

e) Interpret the optimum solution of the primal program for the linear programming problem. (3 *marks*)

QUESTION FOUR (20 MARKS)

(a) Outline the format of a linear programming model for a transportation problem.

(2 *marks*)

(b) A cement manufacturing company intends to meet the demands 50, 70, 40, 20 in thousand bags at various destinations from supplies 60, 20, 100 in thousand bags from various sources. The transport cost per unit amount (thousand Kenya shillings) over the various routes are as given in the matrix below:

$$C = \begin{bmatrix} 3 & 2 & 5 & 7 \\ 4 & 1 & 6 & 1 \\ 8 & 5 & 15 & 9 \end{bmatrix}$$

The company wants to meet the demand at destinations by transporting the cement at the cheapest cost possible. Using the northwest corner (NWC) rule, do the following for the transportation problem:

(i) Determine the optimal solution that minimizes the cost of transport; (16 *marks*)

(ii) Interpret the optimum solution obtained in (i) above. (2 *marks*)

QUESTION FIVE (20 MARKS)

(a) Outline any *four* characteristics of an assignment problem. (4 *marks*)

(b) A courier service company has its fleet of vehicles based at different stations in dispersed locations in the county to transport parcels to various towns. Four vehicles are available to transport parcels from four sources to four towns as destinations. Based on the distance, the amount of fuel in litres each of the vehicles uses from the various sources to the various destinations is as shown in the table below.

		Destination			
		D1	D2	D3	D4
Vehicle	V1	17	20	13	21
	V2	15	21	14	18
	V3	17	18	17	21
	V4	14	22	12	22

The company intends to assign the vehicles to the destinations in such a way that the total fuel consumption by all the vehicles is minimised. Determine the optimal assignment of the vehicles to the destinations that will minimise the total fuel consumption. (16 *marks*)