MACHAKOS UNIVERSITY

University Examinations 2021/2022 Academic Year

SCHOOL OF PURE AND APPLIED SCIENCES

DEPARTMENT OF MATHEMATICS AND STATISTICS

FOURTH YEAR FIRST SEMESTER EXAMINATION FOR

BACHELOR OF SCIENCE IN MATHEMATICS AND COMPUTER SCIENCE

BACHELOR OF SCIENCE IN STATISTICS AND PROGRAMMING

BACHELOR OF SCIENCE IN ACTUARIAL SCIENCE

BACHELOR OF SCIENCE IN MATHEMATICS

SMA 461 OPERATIONS RESEARCH II

DATE: 1/9/2022 **TIME: 2.00-4.00 PM**

INSTRUCTION:

Answer Question One and Any Other Two Questions

You must have a scientific calculator and statistical tables for this paper:

QUESTION ONE (30 MARKS)

(a) (i) Highlight any **two** areas of application of operational research, briefly explaining how it is used in problem solving. (4 *marks*)

(ii) Explain the importance of *queuing theory* as a problem solving technique in operational research, coupled with at least *two* real life examples. (3 *marks*)

(iii) Differentiate between the terms CPA and PERT as used in project network analysis. (3 *marks*)

(iv) Explain two *constraints* which are applicable in the optimisation of a network flow using the maximum flow minimum cut technique. (4 *marks*)

(b) A certain university grades its students using letters A, B, C and D. From past records, it is known that the academic performance as observed in the grades scored by the students has the following probabilities: Grade A is 0.10, Grade B is 0.40, Grade C is 0.35, and Grade D is 0.15. From a random sample of 45 students, the university intends to estimate the expected number of students who will score each grade. The performance of students is random and independent of each other.

Solve the problem using Monte Carlo simulation technique. (8 *marks*)

Use the random numbers below:

98	95	01	81	58	21	61	46	82	39	69	88	80	30	71
02	69	03	11	16	25	06	28	98	38	36	12	73	38	05
10	21	40	76	69	70	27	98	23	74	57	71	61	35	53

(c) A mobile phone network service provider has employed three cashiers based at a customer care desk to serve three counters which offer different customer services. The cashiers take different duration to serve customers, and this also varies between the counters. The average time in minutes each of the cashiers takes to serve a customer at each counter is as shown in the table below.

		Counter		
		C1	C2	C3
Cashier	A	20	15	31
	B	17	16	33
	C	18	19	27

The network provider wishes to assign the cashiers to the counters in such a way that the total time taken to serve customers at the three counters is minimised. Determine the optimal assignment of the cashiers to the counters in such a way that will minimise the total time taken to serve customers. (8 *marks*)

QUESTION TWO (20 MARKS)

(a) Outline *four* characteristics of an *assignment* problem as used in operations research. (4 *marks*)

(b) A mobile phone company sells mobile phones to customers in different regions of the country. The company has four regional sales managers, and it has partitioned the country into four regional blocks A, B, C and D in which the regional sales managers are to be assigned. The managers have different individual potential in sales which varies between the regions. The annual sales revenue in million Kenya shillings by each of the sales managers in each region is as shown in the table below.

		Sales Manager			
		M 1	M 2	M 3	M 4
Region	A	25	18	23	14
	B	38	15	53	23
	C	15	17	41	30
	D	26	28	36	29

The phone company wishes to assign the sales managers to the regions in such a way that the total sales revenue is maximised. Determine the optimal assignment of the sales managers to the regions which will maximise the total sales revenue. (16 *marks*)

QUESTION THREE (20 MARKS)

(a) Outline *four* possible limitations of mathematical *simulation* as an approach to modelling and problem solving in operational research. (4 *marks*)

(b) In a certain donor funded institution, a bank cheque must be signed by four officers: Chief Executive Officer, Chairman, Finance officer and donor representative. The probabilities of the four signatories being available in any given week are as follows: Chief Executive Officer 0.90, Chairman 0.60, Finance Officer 0.95, and donor representative 0.80. Cheques are drawn at random as need arises, and the four signatories are available at random and independent of each other.

Simulate this problem using Monte Carlo method. Hence, taking a period of 15 weeks, determine the expected (mean) number of weeks for which a cheque will be fully signed by all the signatories who are available.

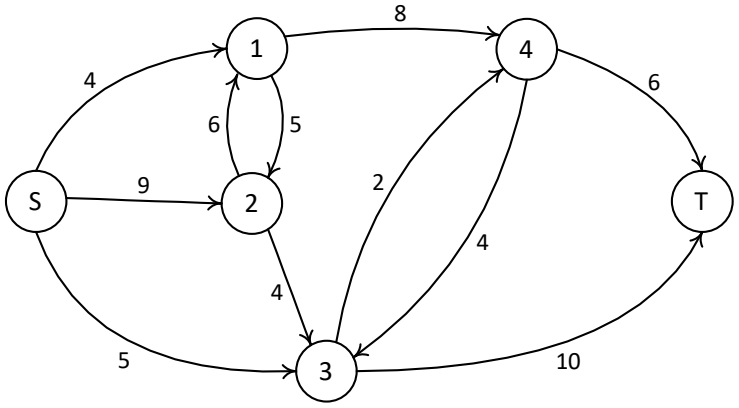
Use the set of random numbers below: (16 *marks*)

CEO	19	20	02	85	02	64	24	85	67	21	95	17	38	76	43
Chairman	21	07	11	94	73	62	36	36	18	14	84	60	90	92	55
Finance Officer	28	85	33	46	16	71	06	28	60	99	09	86	15	60	20
Representative	36	36	81	93	06	88	19	78	41	70	07	38	97	58	31

QUESTION FOUR (20 MARKS)

(a) Outline *three* areas of application in which flow network techniques can be used in problem solving. (2 *marks*)

(b) Given below is a network diagram representing network flows.



Determine the maximum flow and the minimum cut in the flow network. (18 *marks*)

QUESTION FIVE (20 MARKS)

(a) Explain the term *crashing* as used in project network analysis. (2 *marks*)

(b) Machakos University in Collaboration with one Chinese organisation has developed a plan to set up an ICT laboratory. The laboratory project has been broken down into identifiable activities, with their preceding activities and activity duration in multiple time estimates as shown in the precedence table below:

Activity	Preceding Activity	Duration in Days		
		Optimistic	Most Likely	Pessimistic
A	---	06	06	24
B	---	06	12	18
C	A	10	16	34
D	A	07	07	13
E	B	06	12	18
F	B	08	14	20
G	D, E	06	12	18
H	D, E	12	12	30
J	F	06	09	24
K	C, G	08	14	20
L	C, G	06	09	24
M	H, K	06	12	18
N	J	06	06	24
P	L, M	08	14	20
Q	N, P	06	12	18

(i) Construct a network diagram to represent the data in the precedence table. (6 *marks*)

(ii) For each activity, determine the expected duration and standard deviation of the duration. (3 *marks*)

(iii) Determine the critical path of the network and the expected project duration. (4 *marks*)

(iv) Determine the probability of completing the project in 93 days. (3 *marks*)

(v) Determine the *total float* for the longest non-critical chain of activities in the network. (2 *marks*)