MACHAKOS UNIVERSITY

University Examinations for 2021/2022 Academic Year

SCHOOL OF PURE AND APPLIED SCIENCES

DEPARTMENT OF PHYSICAL SCIENCES

FOURTH YEAR FIRST SEMESTER EXAMINATION FOR

BACHELOR OF SCIENCE (APPLIED PHYSICS AND TECHNOLOGY)

BACHELOR OF EDUCATION (SCIENCE)

SPH 402: QUANTUM MECHANICS II

DATE: 26/8/2022 **TIME: 2.00-4.00 PM**

INSTRUCTIONS:

- Question One (1) is **compulsory** (30 marks).
- Answer any **two** questions (each 20 marks).

Use the following constants:

$h = 6.626 \times 10^{-34} m^2 kg/s$, mass of electron=$9.1 \times 10^{-31} kg$, $1ev = 1.6 \times 10^{-19} J$

Useful Identities:

$$\int_0^\infty e^{-ax^2 dx = \frac{1}{2}(\frac{\pi}{a})^{1/2}}$$

$$\int_0^\infty xe^{-ax^2 dx = \frac{1}{2a}}$$

$$\int_0^\infty x^2 e^{-ax^2 dx = \frac{1}{4a}(\frac{\pi}{a})^{1/2}}$$

QUESTION ONE (30 MARKS)

a) i. Identify any two (2) assumptions of quantum mechanics (2 marks)

ii. Explain the meaning of "certain operators commute" (2 marks)

b) Give one example each for the operators that commute and of the operators that do not commute. (2 marks)

c) Calculate the expectation value $\langle p \rangle$ of momentum of a particle trapped in the 1-D box L wide and $\psi(x) = (\frac{2}{L})^{\frac{1}{2}} \sin\left(\frac{n\pi x}{L}\right)$. Hint: $\int \sin(ax)\cos(ax)dx = \frac{1}{2a} sin^2(ax)$. (5 marks)

d) Determine the expectation values $\langle x \rangle$ and $\langle x^2 \rangle$ and hence the uncertainty of a particle in a harmonic oscillator described by the wave equation $\psi(x) = Ne^{-\frac{ax^2}{2}} e^{-\frac{ik_0 t}{\hbar}}$ by first showing that the normalization constant, $N = \left(\frac{a}{\pi}\right)^{\frac{1}{4}}$ (4 marks)

e) Differentiate between the two forms of Zeeman effects (3 marks)

f) State Pauli's exclusion principle (2 marks)

g) Explain the reason why a bound particle has quantized energy values whereas it is not so for a free particle. (2 marks)

h) Find the normalization constant of $A_m e^{im\theta}$ where $0 \leq \theta \leq 2\pi$ (4 marks)

i) An eigen value of an electron confined to a one-dimensional box of length 0.2 nm is 151 eV. Determine the order of excited state (4 marks)

QUESTION TWO (20 MARKS)

a) Show that the commutation relations for the angular momentum operators is expressed in the final form:

$$[\hat{L}_x, \hat{L}_y] = i\hbar\hat{L}_z.,$$

$$[\hat{L}_y, \hat{L}_z] = i\hbar\hat{L}_x.,$$

$$[\hat{L}_z, \hat{L}_x] = i\hbar\hat{L}_y.$$

where orbital angular momentum can be expressed in matrix form as

$\hat{\mathrm{L}} = (\hat{y}\hat{p}_z - \hat{z}\hat{p}_y)\mathrm{i} + (\hat{z}\,\hat{p}_x - \hat{x}\,\hat{p}_z)\hat{\mathrm{j}} + (\hat{x}\,\hat{p}_y - \hat{y}\,\hat{p}_x)\hat{\mathrm{k}}$. (6 marks)

b) The state of a free particle of mass m in 1-D is described by the following quantum wave function

$$\psi(x) = \begin{cases} 0 & x < -a \\ A & -a \leq x \leq 3a \\ 0 & x > 3a \end{cases}$$

Where A is a positive constant

i. Determine A using the normalization condition (3 marks)

ii. Determine the probability that a measurement of the particle's position will reveal it to be in the interval [0, a] (3 marks)

iii. Calculate $\langle x \rangle$ and $\langle x^2 \rangle$ and hence the uncertainty, Δx, for this state (6 marks)

iv. Write down the Schrodinger's time independent equation for $\psi(x)$. (Hint: Remember, it is a free particle). (2 marks)

QUESTION THREE (20 MARKS)

a) Consider a particle of mass m in a 1-D box (particle in an infinite potential well) of length L whose solution to the S-E is given by $\psi(x) = A sin\, kx + B cos\, kx$, show that the energy eigen value is given by $E_n = \frac{n^2 h^2}{8mL^2}$ (14 marks)

b) Calculate the minimum magnetic field needed for the Zeeman effect to be observed in a spectrum line of 360 nm wavelength when the spectrometer whose resolution is 0.02 nm is used. (6 marks)

QUESTION FOUR (20 MARKS)

a) Obtain the energy values of harmonic oscillator by the WKB method. (10 marks)

b) Consider two operators $\hat{A}$ and $\hat{B}$

i. Prove the following commutation relation

$[\hat{A}^2, \hat{B}] = \hat{A}\,[\hat{A}\,,\hat{B}] + [\hat{A}\,,\hat{B}]\,\hat{A}$ (5 marks)

ii. Use the above result and the commutator $[\hat{x}\,,\hat{p}] = i\hbar$ to show that

$[\hat{p}^2, \hat{x}^2] = -2i\hbar(\hat{p}\hat{x} + \hat{x}\hat{p})$ (5 marks)

QUESTION FIVE (20 MARKS)

a) Describe the physical significance of the zero-point energy in a local harmonic oscillator. (4 marks)

b) To capture the spherical symmetry of the three-dimensional dynamics, the orbital angular momentum operators can be expressed in spherical polar coordinates (r, θ, ϕ).

i. Write down the orbital angular momentum operators L_x, L_y, L_z in spherical polar coordinates (8 marks)

ii. Use the results to determine $L^2 = L_x^2 + L_y^2 + L_Z^2$ in spherical polar coordinates.

(8 marks)