MACHAKOS UNIVERSITY

University Examinations for 2021/2022 Academic Year

SCHOOL OF PURE AND APPLIED SCIENCES

DEPARTMENT OF PHYSICAL SCIENCES

FOURTH YEAR FIRST SEMESTER EXAMINATION FOR

BACHELOR OF EDUCATION (SCIENCE)

SPH 400: CLASSICAL MECHANICS

DATE: 22/8/2022 **TIME: 11.00-1.00 PM**

INSTRUCTIONS:

1) **The paper consists of Five questions**
2) **Question ONE is Compulsory and carries Thirty Marks**
3) **Choose any TWO other questions from the paper**

CONSTANTS

Stokes law: $\oint \boldsymbol{F}.\boldsymbol{dr} = \int_s (\boldsymbol{\nabla} \times \boldsymbol{F}) dS$

QUESTION ONE (30 MARKS)

a) Define inertial and non-inertial frames of references (3 marks)

b) State the Lorentz transformation equations and express them in matrix form. (3 marks)

c) Given that the Hamilton H is related to the Lagrangian L through

$$H(q,p,t) = \sum_i \dot{q}_i p_i - L(q,\dot{q},t)$$

where other symbols have usual meaning, obtain the canonical equations of Hamilton.

(5 marks)

d) Calculate the potential function for the vector field, $\mathbf{F} = 2xy^3z^4\,\mathbf{i} + 3x^2y^2z^4\mathbf{j} + 4x^2y^3z^3\mathbf{k}$

(4 marks)

e) A particle of mass m moves under a force $\boldsymbol{F} = -cx^3$, where c is a positive constant.

i. Find the potential energy function; (3 marks)

ii. If the particle starts from rest at $x = -a$, what is its velocity when it reaches $x = 0$?

(2 marks)

iii. Where in the subsequent motion does it come to rest? (2 marks)

f) A simple pendulum has a bob of mass m_1 with a mass m_2 at the moving support (pendulum with moving support) which moves on a horizontal line in the vertical plane in which the pendulum oscillates as shown in Figure 1. Find

i. the Lagrangian of the system (4 marks)

ii. the lagrange's equations of motion (4 marks)

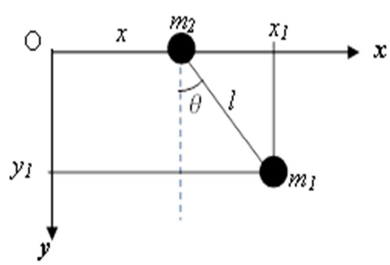

Figure 1

QUESTION TWO (20 MARKS)

a) Define with an illustration the term canonical transformation. (3 marks)

b) For the Atwood's machine shown in Figure 2 the pulley rotates as the masses m_1 and m_2 move. The length of the string is l and r is the radius of the pulley. Calculate

i. the Lagrangian of the system. (4 marks)

ii. the acceleration of the masses. (4 marks)

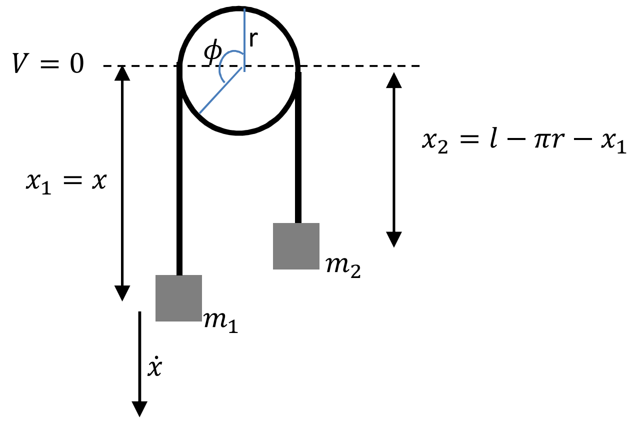

Figure 2

c) A 1 kg and 3kg masses move under a force such that their position vectors at time t are $\mathbf{r_1} = 2\,\hat{\mathbf{i}} + 4t^2\hat{\mathbf{k}}$, $\mathbf{r_2} = 4t\hat{\mathbf{i}} - \hat{\mathbf{k}}$, and $\mathbf{r_3} = (\cos \pi t)\hat{\mathbf{i}} + (\sin \pi t)\hat{\mathbf{j}}$, respectively. Calculate the angular momentum of the system about the origin at t = 1 s.

(5 marks)

d) If the Hamiltonian of a system is $H = \frac{p^2}{2} - \frac{1}{2q^2}$, show that $F = \frac{pq}{2} - Ht$ is a constant of motion. (4 marks)

QUESTION THREE (20 MARKS)

a) Explain holonomic and non-holonomic constraints, giving two examples of each. (4 marks)

b) A block of mass m_2 is sliding on a wedge of mass m_1. The wedge is sliding on a horizontal table as shown in Figure 3. Calculate

i. The Lagrangian function. (4 marks)

ii. The equation of motion of the system. (4 marks)

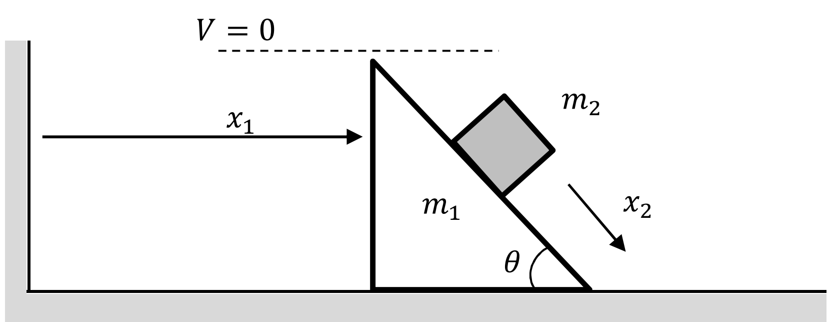

Figure 3

c) A mass m is suspended by a massless spring of spring constant k. The suspension point is pulled upwards with constant acceleration a_0. Calculate

i. the Hamiltonian of the system, (4 marks)

ii. Hamilton's equations of motion (4 marks)

QUESTION FOUR (20 MARKS)

a) Express D' Alembert's principle in the integral form. What is its advantage over the one in the differential form? (3 marks)

b) State and explain the Euler-Lagrange differential equation in the calculus of variation. (3 marks)

c) Using Lagrange's method of undetermined multiplier, find the equation of motion and force of constraint in the case of a simple pendulum. (8 marks)

d) A particle of mass *m* is placed at the top of a smooth hemisphere of radius *a*. Find the reaction of the hemisphere on the particle. If the particle is disturbed, at what height does it leave the hemisphere? (6 marks)

QUESTION FIVE (20 MARKS)

a) Calculate the equations of motion of a pendulum bob suspended by a spring and allowed to swing in a vertical plane, given that (6 marks)

$$L = \frac{1}{2}m(\dot{r}^2 + r^2\dot{\theta}^2) + mgr\cos\theta - \frac{1}{2}k(r - r_o)^2$$

b) The length of a spaceship is measured to be exactly half its proper length. What is

i. the speed of the spaceship relative to the observer on earth? (3 marks)

ii. the dilation of the spaceship's unit time? (3 marks)

c) An inertial frame S moves with respect to another inertial frame S' with a uniform velocity 0.6 c along the x, x'-axes. The origins of the two systems coincide when $t = t' = 0$. An event occurs at $x_1 = 10$ m, $y_1 = 0, z_1 = 0$, $t_1 = 2 \times 10^{-7}$ s. Another event occurs at $x_2 = 40$ m, $y_2 = 0, z_2 = 0, t_2 = 3 \times 10^{-7}\,s$ in S',

i. what is the time difference? (4 marks)

ii. what is the distance between the events? (4 marks)