MACHAKOS UNIVERSITY

University Examinations for 2021/2022 Academic Year

DIRECTORATE OF TVET

SECOND YEAR FIRST TERM EXAMINATION FOR

DIPLOMA IN BUILDING TECHNOLOGY

DIPLOMA IN CIVIL ENGINEERING

MATHEMATICS 11

DATE: 22/7/2022 **TIME: 8.30-11.30 AM**

INSTRUCTIONS

You should have the following for this examination:

Mathematical tables/ Non programmable scientific calculator

Answer any five questions in the answer booklet provided.

All questions carry equal marks.

1. a) Differentiate from the first principle $y = x^2 + 4x$ (5 marks)

   b) Find $\frac{dy}{dx}$ given that $y = \sec^3 x$ (5 marks)

   c) Compute the derived function given that $y = \frac{x^2}{1 + x^3}$ (5 marks)

   d) Given $x^2 y - x^5 y^3 + 2 = 0$ evaluate $\frac{dy}{dx}$ at the point $(1,2)$ (5 marks)

2. a) Given that $I_n = \int x^n e^x dx$, deduce the reduction formula hence find $\int x^4 e^x dx$ (10 marks)

   b) The area bounded by a curve $y = 4x - x^2$ and the x-axis is rotated through one revolution about x-axis;

      i. Sketch the curve.

ii. Calculate the volume generated. (10 marks)

3. a) Given the function $f(x, y) = x\cos y + ye^x$, find $\frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2}$ (5 marks)

b) A curve is defined by parametric equations $x = \theta - \sin 2\theta$, $y = 1 + \cos 2\theta$. Determine the radius of curvature at the point where $\theta = \frac{\pi}{3}$ radians. (9 marks)

c) Determine the values of x for which the function $f(x) = x^4 - 2x^2$ has a maxima and minima. (6 marks)

4. a) Solve the differential equation $\frac{d^2y}{dx^2} - 2\frac{dy}{dx} - 3y = 36e^{5x}$ when $x = 0$, $y = 9$ and $\frac{dy}{dx} = 25$ (12 marks)

b) Solve the differential equation $\frac{dy}{dx} = y^2 - 4$ given that when $x = 0$, $y = 1$ (8 marks)

5. a) Integrate the following;

i. $\int_0^{\pi} x^2 \cos 4x \, dx$

ii. $\int \frac{3x + 11}{x^2 - x - 6} dx$ (15 marks)

b) Determine the region bounded by $y = 2x^2 + 10$ and $y = 4x + 16$ (5 marks)

6. a) Show that $\frac{1}{y}\frac{\partial z}{\partial x} = \frac{1}{x}\frac{\partial z}{\partial y}$ given that $z = \sin xy$ (5 marks)

b) Show that the function $z = x^2 + y^2$ has one stationary point only and determine its nature. (7 marks)

c) An open rectangular container is to have a volume of $32M^3$. Determine the dimensions and the total surface area so that the total surface area is minimum. (8 marks)

7. a) Find the coordinates of the centroid of the area bounded by the curve $y = 3x^2$ and $x = 0 \, and \, x = 4$ (6 marks)

b) Use the implicit rule of differentiation to determine the equations of the;

i. Tangent

ii. Normal to the curve

Given that $9x^2 + 3y^2 - 4xy + 6x - 8y = 6$ at the point $(1,3)$ (8 marks)

c) Determine the first and second partial derivatives given $f(s,t) = s^2t + \ln(t^2 - s)$

(6 marks)