



# MACHAKOS UNIVERSITY

University Examinations for 2022/2023 Academic Year

SCHOOL OF BUSINESS, ECONOMICS AND HOSPITALITY AND TOURISM

MANAGEMENT

DEPARTMENT OF ECONOMICS

SECOND YEAR FIRST SEMESTER EXAMINATION FOR

BACHELOR OF ECONOMICS AND FINANCE

BACHELOR OF ECONOMICS AND STATISTICS

BACHELOR OF ECONOMICS

EES 200: MATHEMATICS FOR ECONOMICS II

DATE:

TIME:

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## INSTRUCTIONS

- (i) Answer question one (COMPULSORY) and any other two questions
- (ii) Do not write on the question paper
- (iii) Show your workings clearly

## QUESTION ONE (COMPULSORY) (30 MARKS)

- a) Determine the equilibrium prices and quantities in the following two problems:
- i)  $Q_d = 3 - P^2$ ,  $Q_s = 6P - 4$  (2 marks)
  - ii)  $Q_d = 8 - P^2$ ,  $Q_s = P^2 - 2$  (2 marks)
- b) Prove Euler's Theorem given the following production functions:
- i)  $Q = 20K^{0.3}L^{0.4}$  (3 marks)
  - ii)  $Q = 50K^{0.4}L^{0.6}$  (3 marks)
- c) The following table gives some statistics of 3 sectors in an economy:

		Input to				
		Agriculture	Manufacturing	Services	Final demand	Total Output
Output From	Agriculture	150	225	125	100	600
	Manufacturing	210	250	140	300	900
	Service	170	0	30	100	300
Other inputs		70	425	5		
Total inputs		600	900	300		

**Required:**

- i) The input-output matrix (2 marks)
- ii) The Leontief Matrix (2 marks)
- iii) The output requirement for each sector given that the final demand changes to  $D = \begin{pmatrix} 200 \\ 600 \\ 200 \end{pmatrix}$  (6 marks)
- d) The initial value of wine is  $W_0 = 100$ , the rate of growth is 3%, and the growing value of wine is given by the following equation.  
 $W_t = W_0 e^{rt}$   
Required: Calculate the value of wine at the end of the 9th year. (3 marks)
- e) Find the producer's and consumer's surplus given the following functions: (7 marks)
- $$\begin{array}{ll} P = 3 + Q^2; & P_e = 19 \\ P = 80 - 4Q & P_e = 10 \end{array}$$

**QUESTION TWO (20 MARKS)**

- a) Assuming that the rate of net investment flow is given by the following function and that  $K(0) = 60$ :

$$I(t) = 100t^{\frac{1}{3}}$$

- i) Find the time path of capital stock  $K(t)$  (2 marks)
- ii) Find the amount of capital formation over the interval  $[1,6]$  (3 marks)
- b) Given the following national income model, compute the equilibrium values of the endogenous variables using the matrix inversion method: (8 marks)

$$Y = C + I + G$$

$$C = 180 + 0.7Y^d$$

$$T = 70$$

$$I = 60 + 0.1Y$$

$$G = 80$$

- c) Prove Young's theorem for the following functions:
- i)  $Q = 3K^3 - 5KL^2 - L^3$  (2 marks)
- ii)  $Y = [18X_1^2 + 3X_2 X_1 - X_2^2]^4$  (3 marks)
- d) The marginal revenue function for a firm is given as:

$$MR = 20 - 50Q$$

Compute the total revenue function (2 marks)

### QUESTION THREE (20 MARKS)

- a) Given the following constrained optimization problem:

$$\text{Optimize } Z = x^2 + y^2$$

$$\text{Subject to } 5x + 2y = 25$$

Required:

- i. The critical values of  $x, y$  and  $\lambda$  (6 marks)
- ii. Test whether the critical values yield a maximum or minimum point (6 marks)

- b) A production function (total product) is given as:

$$Q = 120K^{0.4}L^{0.5}$$

Determine:

- (i) The marginal product of labour and marginal product of capital (3 marks)
- (ii) The slope of the isoquant (2 marks)
- (iii) The nature of the marginal products (3 marks)

### QUESTION FOUR (20 MARKS)

- a) Evaluate the following

i)  $\int 2x \cdot (x^2 + 1) dx$  (4 marks)

ii)  $\int_2^3 \frac{3x^2}{x^3+3} \cdot dx$  (3 marks)

- b) The production function for a firm is given as :  $Q = 10L^{1/2}K^{1/2}$

If each unit of labour used costs \$5 while each unit of capital cost \$3:

- (i) Find the units of labour and capital to be used so as to minimize the production cost given that the firm wants to produce 250 units of output. (6 marks)
- (ii) Do these optimal values represent a maxima or minima? Prove your answer. (7 marks)

### QUESTION FIVE (20 MARKS)

- a) The average revenue (AR) of coffee harvested from a plantation is growing at 5% per annum, while the quantity (Q) supplied is growing at 2% per annum. Calculate the annual growth rate of the total revenue (TR) from the sale of the tea. (3 marks)

- b) Given the following matrix:

$$A = \begin{bmatrix} 4 & 9 & 10 \\ 5 & 2 & 0 \\ 6 & 1 & 6 \end{bmatrix}$$

Compute

- i. The inverse of the matrix (5 marks)
- ii. The rank of the matrix (5 marks)

c) A demand function and other variables are given as:

$$Q_A = 180 - 4P_A + 0.8P_B + 0.2Y$$

$$P_A = 12 \quad P_B = 20 \quad Y = 200$$

Comment on the following statements and prove your answer

- (i) Good A is an inferior good (4 marks)
- (ii) Good A and B are not related (3 marks)