MACHAKOS UNIVERSITY

University Examinations for 2022/2023 Academic Year

SCHOOL OF PURE AND APPLIED SCIENCES

DEPARTMENT OF MATHEMATICS AND STATISTICS

THIRD YEAR FIRST SEMESTER EXAMINATION FOR

BACHELOR OF SCIENCE (MATHEMATICS & COMPUTER SCIENCE)

BACHELOR OF SCIENCE (MATHEMATICS)

SMA 302: GROUP THEORY

DATE:15/12/2022 **TIME:8.30 – 10.30A.M**

INSTRUCTIONS TO CANDIDATES

Answer questions ONE and ANY TWO Questions

QUESTION ONE (30 MARKS) COMPULSORY

a) Let G be the group of all real 2×2 matrices $\begin{pmatrix} a & b \\ c & d \end{pmatrix}$ such that $ad - bc \neq 0$ under multiplication. Let $k = \left\{ \begin{pmatrix} 1 & b \\ 0 & 1 \end{pmatrix} \middle| \, b \in \mathbb{R} \right\}$, show that $k < G$ (k is a subgroup of G) (5 marks)

b) Construct the multiplication table for A_3. (5 marks)

c) Let $G = Z_4 = \{0,1,2,3\}$, the group of integers modulo 4 under addition. Determine the order of 0, 1, 2 and 3 (5 marks)

d) Prove that every cyclic group is abelian (5 marks)

e) Let $G = \{\pm 1, \pm i, \pm j, \pm k\}$, the quarternion group. Determine all the left cosets of $H = \{1, -1\}$ in G (5 marks)

f) If $f: G \to G'$ is an isomorphism of G with G' and e is the identity of G then prove that $f(e)$ is the Identity in G'. (5 marks)

QUESTION TWO 20 MARKS

a) State and proof Lagrange's theorem (8 marks)

b) Define a group (5 marks)

c) If A is a non empty set and S_A a collection permutations of A. Then prove that S_A is a group under permutation multiplication. (7 marks)

QUESTION THREE 20 MARKS

a) Prove that every group of prime order is cyclic (5 marks)

b) Prove that the collection of all even permutations of a finite set of n elements form a subgroup of order $\frac{n!}{2}$ (5 marks)

c) Prove that if G is a group and $a \in G$, then $H = \{a^n | n \in Z\}$ is a subgroup of G and is the smallest subgroup which contains a. (5 marks)

d) Find all the sylow 3-subgroups of S_4 and demonstrate that they are conjugate. (5 marks)

QUESTION FOUR 20 MARKS

a) Prove that every subgroup of an abelian group is a normal subgroup (5 marks)

b) Prove that if G is a group with binary operation * then the right and the left cancellation holds in G. . (5 marks)

c) Define a function $f: G \to G^1$ by $f(x) = axa^{-1}$ show that f is isomorphism. (5 marks)

d) Prove that the centre of a group is a normal subgroup of the group. (5 marks)

QUESTION FIVE 20 MARKS

a) Let $G = S_3$ and consider the subgroup $H = \langle(13)\rangle = \{1, (13)\}$, determine all the conjugates of H
(6 marks)

b) Prove that in a group G with the operation * , there is only one identity and inverse. (6 marks)

c) State whether the following permutation is odd or even

i) $(13245)(679)(1011)$ (4 marks)

ii) $(134)(587)(26)$ (4 marks)