MACHAKOS UNIVERSITY

University Examinations for 2022/2023 Academic Year

SCHOOL OF PURE AND APPLIED SCIENCES

DEPARTMENT OF MATHEMATICS AND STATISTICS

SECOND YEAR FIRST SEMESTER EXAMINATION FOR

BACHELOR OF SCIENCE (ACTUARIAL MATHEMATICS I)

EDUCATION (SCIENCE)

BACHELOR OF EDUCATION (ARTS)

SAC 203: ACTUARIAL MATHEMATICS I:

DATE:25/7/2023 **TIME:11:00-1:00 P.M**

INSTRUCTIONS: Answer question one and any other two questions

QUESTION ON (COMPULSORY) (30 MARKS)

a) Define the following notations.

i. ${}_nd_x$. (2 marks)

ii. L_x. (2 marks)

b) Suppose that a fund initially containing $1000 accumulates with a force of interest $d(t) = 1/(1 + t)$, for $t > 0$. What is the value of the fund after 5? years? (3 marks)

c) A population is subject to a constant force of mortality of 0. 015.Calculate:

i. The probability that a life aged 20 exact will die before age 21.25 exact. (3 marks)

ii. The curtate expectation of a life aged 20 exact. (5 marks)

d) Show that:

$\bar{a}_{x|y} = \bar{a}_y - \bar{a}_{xy}$ (5 marks)

e) Calculate the following probabilityy on the basis of English Life Table No. 15-Males. the probability that a person aged 60 dies within the first five years after retiring at age 65. (5 marks)

f) For a force of mortality μ_x that is known to follow Gompertz' law, calculate the parameters B and C if $\mu_{50} = 0.017609$ and $\mu_{55} = 0.028359$ (5 marks)

QUESTION TWO (20 MARKS)

An investigation was carried out into the effects of lifestyle factors on the mortality of people aged between 50 and 65 years. The investigation took the form of a prospective study following a sample of several hundred individuals from their 50th birthdays until their 65th birthdays and collecting data on the following covariates for each person:

X_1 Sex (a categorical variable with 0 = female, 1 = male)

X_2 Cigarette smoking (a categorical variable with 0 = non-smoker, 1 = smoker)

X_3 Alcohol consumption (a categorical variable with 0 = consumes fewer than 21 units of alcohol per week, 1 = consumes 21 or more units of alcohol per week)

In addition, data were collected on the age at death for persons who died during the period of investigation.
In order to analyse the data, it was decided to use a Gompertz hazard, $\lambda_x = Bc^x$, where x is the duration since the start of the observation.

a) Explain why the Gompertz hazard might be appropriate for analysing the mortality of persons aged between 50 and 65 years. (2 marks)

b) Show that the substitution:

$$B = \exp(\beta_0 + \beta_1 X_1 + \beta_2 X_2 + \beta_3 X_3),$$

in the Gompertz model (where $\beta_0, \ldots \beta_3$ are parameters to be estimated), leads to a proportional hazards model for this particular analysis. (3 marks)

c) Using the Gompertz hazard, the parameter estimates in the proportional hazards model were as follows:

Covariate	*Parameter estimate*	*Parameter*
Sex	β_1	+0.40
Cigarette smoking	β_2	+0.75
Alcohol consumption	β_3	−0.20
	β_0	−5.00
	c	+1.10

i) Describe the characteristics of the person to whom the baseline hazard applies in this model. (2 marks)

ii) Calculate the estimated hazard for a female cigarette smoker aged 55 years who do not consume alcohol. (3 marks)

iii) Show that, according to this model, a cigarette smoker at any age has a risk of death roughly equal to that of a non-smoker aged eight years older.
. (10 marks)

QUESTION THREE (20 MARKS)

A pension scheme provides the following benefit to the spouse of a member, following the death of the member in retirement:
A pension of £10,000 *pa* payable during the lifetime of the spouse, but ceasing 15 years after the death of the member if that is earlier. All payments are made on the anniversary of the member's retirement.
Calculate the expected present value of the spouse's benefit in the case of a female member retiring now on her 60th birthday, who has a husband aged exactly 55.
Basis: PA92C20 mortality, 4% *pa* interest .

QUESTION FOUR (20 MARKS)

T_x denotes the future lifetime of a life aged x .

a) Write down the probability density function of T_x . (1 mark)

b) Using your answer to (a), show that (10 marks)

i. $\frac{\partial}{\partial s}\log\ {}_sp_x = -\mu_{x+s}$ and,

ii. ${}_tp_x = \exp\{-\int_0^t \mu_{x+s}\ ds\}$

c) In a certain population, the force of mortality is given by,

	μ_x
$60 < x \leq 70$	0.01
$70 < x \leq 80$	0.015
$x > 80$	0.025

Calculate the probability that a life aged exactly 65 will die between exact ages 80 and 83. (9 marks)

QUESTION FIVE (20 MARKS)

The force of interest, $\delta(t)$, is a function of time and at any time t, measured in years, is given by the formula

$$\delta(t) = \begin{cases} 0.05 + 0.001t & 0 \le t \le 20 \\ 0.05 & t > 20 \end{cases}$$

a

a) Derive and simplify as far as possible expressions for $v(t)$, where $v(t)$ is the present value of a unit sum of money due at time t. (6 marks)

b) Calculate:

i. the present value of £100 due at the end of 25 years. (4 marks)

ii. the rate of discount per annum convertible quarterly implied by the transaction in part (b)(i). (3 marks)

c) A continuous payment stream is received at rate $30e^{-0.015t}$ units per annum between t = 20 and t = 25. Calculate the accumulated value of the payment stream at time t = 25. (7 marks)