MACHAKOS UNIVERSITY

University Examinations for 2022/2023 Academic Year

SCHOOL OF PURE AND APPLIED SCIENCES

DEPARTMENT OF MATHEMATICS AND STATISTICS

SECOND/THIRD YEAR FIRST SEMESTER EXAMINATION FOR

BACHELOR OF SCIENCE (STATISTICS AND PROGRAMMING)
BACHELOR OF SCIENCE (ACTUARIAL SCIENCE)
BACHELOR OF SCIENCE (MATHEMATICS)
BACHELOR OF EDUCATION

SST 201, SMA 362 : OPERATIONS RESEARCH I

DATE: **TIME:**

<u>*Instructions to the Candidate:*</u>

1. *Answer* **Question 1** *and any other* **two** *questions.*
2. *You must have a Scientific Calculator and Graph Paper.*

QUESTION ONE (COMPULSORY) (30 MARKS)

(a) Explain briefly *four* conditions which must be satisfied for a problem to be solved using linear programming technique. (4 marks)

(b) Explain each of the following types of variables as used in linear programming, in the context of both an initial feasible solution and an optimum solution:

(i) Slack variable;
(ii) Surplus variable;
(iii) Artificial variable. (6 marks)

(c) Given the linear programming model below:

Minimise : $z = 10x_1 + 8x_2 + 15x_3$

Subject to : $3x_1 + 2x_2 + 5x_3 \geq 30$

$7x_1 + 4x_2 + 6x_3 \geq 45$

$2x_1 + 5x_2 + 3x_3 \geq 60$

With : $x_1, \ x_2, \ x_3 \geq 0$

Determine its symmetrical dual program. (2 marks)

(d) A lamp production plant produces two products – florescent tubes and energy saving bulbs. Tubes are sold at a price of Ksh 60 per unit while bulbs are sold at a price of Ksh 80 per unit. The manager of the firm wants to determine the daily production plan which maximises contribution to sales revenue. The production constraints are as shown in the table below:

	Labour hours	Machine hours	Glass material	Coating material
Tubes	3	5	25	1
Bulbs	5	4	18	1
Maximum available	900	1200	4500	200

The Ministry of Energy has issued a directive that a minimum of 40 bulbs must be produced per day in conformity with an energy saving strategy.

(i) Formulate a linear programming model for the problem. (3 *marks*)

(ii) Using the graphical method, determine the optimum production plan which maximises the daily sales revenue for the bottling firm. (6 marks)

(iii) Interpret the optimum solution obtained in (ii) above. (3 marks)

(e) A shipping company is required to meet the demands at various destinations by transporting 80, 100, 140, 60, 120 containers respectively from various supply sources with 120, 180, 70, 130 containers. The transport cost in thousand Kenya shillings per unit of container over the various routes are as given in the matrix below:

$$C = \begin{bmatrix} 4 & 8 & 7 & 2 & 6 \\ 8 & 6 & 5 & 4 & 2 \\ 6 & 2 & 1 & 3 & 5 \\ 5 & 1 & 2 & 4 & 3 \end{bmatrix}$$

Sources: (120, 180, 70, 130)

Destinations: (80, 100, 140, 60, 120)

The company wants to meet the demand at the destinations by transporting the containers at the cheapest cost possible. Derive the *initial basic solution* and its corresponding total cost of transport using each of the following methods:

(i) Northwest corner (NWC) rule;

(ii) Vogel's approximation method (VAM). (6 marks)

QUESTION TWO (20 MARKS)

A restaurant sells three types of hot drinks – tea, cocoa and coffee. The sale prices per cup of the drinks are: tea Ksh 80, cocoa Ksh 50 and coffee Ksh 100. The daily production plan for the sales revenue has been modelled as an LP program as shown below.

Maximise :	$z = 80x_1 + 50x_2 + 100x_3$	
Subject to :	$2x_1 + 3x_2 + x_3 \leq 4000$	Machine hours
	$x_1 + x_3 \leq 1500$	Labour hours
	$2x_1 + 4x_3 \leq 2000$	Colouring agent
	$x_2 \leq 500$	Comesa trade agreement
With :	$x_1 \geq 0,\ x_2 \geq 0,\ x_3 \geq 0$	Non-negativity conditions

where x_1 - the number of cups of tea produced,
x_2 - the number of cups of cocoa produced, and
x_3 - the number of cups of coffee produced.

The restaurant wishes to determine the daily production plan which maximises contribution to its sales revenue.

(i) Using the Simplex method, determine the optimum daily production plan for the restaurant. (16 marks)

(ii) Interpret the optimum solution obtained in (i) above. (4 marks)

QUESTION THREE (20 MARKS)

Below is a linear programming model for a given problem. Use it to answer the questions that follow.

Minimise : $z = 120x_1 + 140x_2 + 80x_3$

Subject to : $x_1 + 2x_2 + x_3 \geq 30$

$2x_1 + x_2 + x_3 \geq 45$

With : $x_1,\ x_2,\ x_3 \geq 0$

(i) Explain the rationale for the use of *duality* in solving a linear programming problem using the simplex technique. (2 marks)

(ii) Determine the *symmetrical dual* of the linear programming model above. (2 marks)

(iii) Using the Simplex method, determine the optimum solution of the dual program for the linear programming model above. (9 marks)

(iii) Using the relationship between a primal program and its symmetrical dual program, extract the optimum solution of the primal program from the optimum solution of its dual program. (4 marks)

(iv) Interpret the optimum solution of the primal program for the linear programming problem. (3 marks)

QUESTION FOUR (20 MARKS)

(a) Explain the *two* major components of a linear programming model. (4 marks)

(b) Outline *four* characteristics of an assignment problem as used in operations research. (4 marks)

(c) A mobile phone company sells mobile phones to customers in different regions of the country. The company has four regional sales managers, and it has partitioned the country into four regional blocks A, B, C and D, in which the regional sales managers are to be assigned. The managers have different individual potential in sales which varies between the regions. The annual sales revenue in million Kenya shillings by each of the sales managers in each region is as shown in the table below.

		Sales Manager			
		M 1	M 2	M 3	M 4
Region	A	25	18	23	14
	B	38	15	53	23
	C	15	17	41	30
	D	26	28	36	29

The phone company wishes to assign the sales managers to the regions in such a way that the total sales revenue is maximised. Determine the optimal assignment of the sales managers to the regions which will maximise the total sales revenue. (12 marks)

QUESTION FIVE (20 MARKS)

A cement distributor is required to meet the demands 50, 70, 100, 60 in thousand bags at various destinations from supplies 100, 60, 120 in thousand bags from various sources. The transport cost per unit amount (thousand Kenya shillings) over the various routes are as given in the matrix below:

$$C = \begin{bmatrix} 5 & 1 & 3 & 4 \\ 2 & 4 & 2 & 5 \\ 1 & 3 & 2 & 4 \end{bmatrix}$$

The company wants to meet the demand at destinations by transporting the cement at the cheapest cost possible. Determine the optimal solution that minimizes the cost of transport using each of the following methods:

(i) the northwest corner (NWC) rule;

(ii) the Vogel's approximation method (VAM); (17 marks)

(iii) Interpret the optimum solution obtained in (i) above. (3 marks)