MACHAKOS UNIVERSITY

University Examinations for 2022/2023 Academic Year

SCHOOL OF PURE AND APPLIED SCIENCES

DEPARTMENT OF MATHEMATICS AND STATISTICS

THIRD YEAR FIRST SEMESTER EXAMINATIONS FOR

BACHELOR OF SCIENCE (MATHEMATICS AND STATISTICS)

BACHELOR OF SCIENCE (ANALYTICAL CHEMISTRY)

BACHELOR OF SCIENCE (COMPUTER SCIENCE)

BACHELOR OF SCIENCE (MATHEMATICS)

BACHELOR OF EDUCATION (SCIENCE)

SMA 301: REAL ANALYSIS II

DATE: **TIME:**

INSTRUCTIONS

1. Answer **question one** and **any other two** questions
2. Write the name of the unit and registration number on each page of your answer sheet.

QUESTION ONE (COMPULSORY)(30 MARKS)

a) Differentiate between uniform convergence and Cauchy condition for uniform convergence (4 marks)

b) Prove that if a sequence (f_n) of continuous functions $f_n : A \to \mathbb{R}$ converges uniformly on $A \subset \mathbb{R}$ to $f : A \to \mathbb{R}$, then f is continuous of A. (6 marks)

c) Prove that a monotonic function f on $[a,b]$ is a function of bounded variation on $[a,b]$ and $V(f) = |f(b) - f(a)|$ (5 marks)

d) For what values of x does the series $\sum_{n=1}^{\infty} \frac{x^{n-1}}{n3^n}$ (5 marks)

e) Let $f(x) = x^2 + 3 \; and \; g(x) = \begin{cases} 0 & , x = 0 \\ 1 + x^3 & , 0 < x \le 1 \end{cases}$. Evaluate $\int_0^1 fdg$ (5 marks)

f) Suppose f is Riemann integrable function on $[a,b]$, then show that $|f|$ is Riemann integrable on $[a,b]$ and $\left|\int_a^b f\right| \le \int_a^b |f|$ (5 marks)

QUESTION TWO (20 MARKS)

a) Let $f(x) = \begin{cases} 0, 0 \le x \le 1 \\ 1, 1 < x \le 2 \end{cases} and \; g(x) = \begin{cases} 0, 0 \le x < 1 \\ 1, 1 \le x \le 2 \end{cases}$ Show that f is integrable with the respect g and find $\int_0^2 fdg$ (6 marks)

b) Let f be a function of bounded variation on $[a,b]$ and $|f(x)| \ge M \;\; \forall x \in [a,b]$ for some positive number M. Show that the function $h = \frac{1}{f}$ is a function of bounded variation on $[a,b]$ (4 marks)

c) converges For what values of x does the series $\sum_{n=1}^{\infty} \frac{(-1)^{n-1} x^{2n-1}}{(2n-1)!}$ converges (5 marks)

d) Prove that if f and g are functions of bounded variation on $[a,b]$ then $f.g$ is a function of bounded variation on $[a,b]$ (5 marks)

QUESTION THREE (20 MARKS)

a) Let $f(x) = e^x \; and \; g(x) = \begin{cases} x^2 & , 0 \le x < 1 \\ 1 + 2x & , 1 \le x \le 2 \\ 5 + x & , 2 < x \le 3 \end{cases}$. Evaluate $\int_0^3 fdg$ (5 marks)

b) Suppose f and g are Riemann integrable functions on $[a,b]$ then show that for any number c, $c.f$ is Riemann integrable on $[a,b]$ (6 marks)

c) Define $f_n : \mathbb{R} \to \mathbb{R}$ by $f_n(x) = \frac{x^2}{\sqrt{x^2 + \frac{1}{n}}}$. Determine where f_n converges pointwise on $\mathbb{R}$ (4 marks)

d) Prove that if f and g are functions of bounded variation on $[a,b]$ then $f + g$ is a function of bounded variation on $[a,b]$ and $V(f+g,[a,b]) \le V(f,[a,b]) + (g,[a,b])$ (5 marks)

QUESTION FOUR (20 MARKS)

a) Define $f_n[0,1] \to \mathbb{R}$ by $f_n(x) = \begin{cases} 2n^2 x & if\ 0 \le x \le \frac{1}{2n} \\ 2n^2(\frac{1}{n-x})\, if & \frac{1}{2n} < x < \frac{1}{n} \\ 0 & \frac{1}{n} < x \le 1 \end{cases}$. Determine where $f_n(x)$ converges pointwise and whether it converges uniformly on $[0,1]$ (7 marks)

b) Prove that if f is a continuous real valued function on on $[a,b]$, then f is Riemann integrable on $[a,b]$ (8 marks)

c) Prove that if the derivative $f^{'}$ of f exists and is bounded on $[a,b]$, then f is of bounded variation on $[a,b]$ (5 marks)

QUESTION FIVE (20 MARKS)

a) Prove that if f is a function of bounded variation on $[a,b]$, then f is bounded (5 marks)

b) Prove that a sequence (f_n) of functions $f_n : A \to \mathbb{R}$ converges uniformly on A if and only if it is uniformly Cauchy on A (7 marks)

c) Let $f(x) = \begin{cases} 2x & ,0 \le x < 1 \\ 2x^3 & ,1 < x < \frac{3}{2} \\ e^x & ,\frac{3}{2} < x \le 3 \end{cases}$ *and* $g(x) = \begin{cases} 0\,,0 \le x < 1 \\ 1\,,1 \le x \le 2 \\ 3, 2 < x \le 3 \end{cases}$. Evaluate $\int_0^3 fdg$ (8 marks)