MACHAKOS UNIVERSITY

University Examinations 2022/2023

SCHOOL OF PURE AND APPLIED SCIENCES

DEPARTMENT OF PHYSICAL SCIENCES

FIRST YEAR FIRST SEMESTER EXAMINATION FOR

BACHELOR OF SCIENCE (TELECOMMUNICATION AND INFORMATION TECHNOLOGY)

BACHELOR OF EDUCATION (SPECIAL NEEDS)

BACHELOR OF EDUCATION (SCIENCE)

BACHELOR OF SCIENCE (MATHEMATICS)

SPH 103: MATHEMATICS FOR PHYSCIS

DATE: **TIME:**

INSTRUCTIONS:

- The paper consists of **two** sections.
- Section **A** is **compulsory** (30 marks).
- Answer any **two** questions from section **B** (each 20 marks).

Standard integrals

Integrand	Integral	Integrand	Integral
$\sin x$	$-\cos x$	$\sinh x$	$\cosh x$
$\cos x$	$\sin x$	$\cosh x$	$\sinh x$
$\tan x$	$-\ln(\cos x)$	$\tanh x$	$\ln(\cosh x)$
$\operatorname{cosec} x$	$\ln(\tan \frac{x}{2})$	$\operatorname{cosech} x$	$\ln(\tanh \frac{x}{2})$
$\sec x$	$\ln(\tan x + \sec x)$	$\operatorname{sech} x$	$2\tan^{-1}(e^x)$
$\cot x$	$\ln(\sin x)$	$\coth x$	$\ln(\sinh x)$
$\sec^2 x$	$\tan x$	$\operatorname{sech}^2 x$	$\tanh x$
$\tan x \sec x$	$\sec x$	$\tanh x \operatorname{sech} x$	$-\operatorname{sech} x$
$\cot x \operatorname{cosec} x$	$-\operatorname{cosec} x$	$\coth x \operatorname{cosech} x$	$-\operatorname{cosech} x$

$\frac{1}{\sqrt{a^2 - x^2}}$	$\sin^{-1}\left(\frac{x}{a}\right)$	or	$-\cos^{-1}\left(\frac{x}{a}\right)$
$\frac{1}{\sqrt{x^2 + a^2}}$	$\sinh^{-1}\left(\frac{x}{a}\right)$	or	$\ln\left(x + \sqrt{x^2 + a^2}\right)$
$\frac{1}{\sqrt{x^2 - a^2}}$	$\cosh^{-1}\left(\frac{x}{a}\right)$	or	$\ln\left(x + \sqrt{x^2 - a^2}\right)$
$\frac{1}{x^2 + a^2}$	$\frac{1}{a}\tan^{-1}\left(\frac{x}{a}\right)$		

Standard differentials

y or $f(x)$	$\frac{dy}{dx}$ or $f'(x)$	
K, K is a constant	$\frac{d}{dx}(K)$	$= 0$
x^n	$\frac{d}{dx}(x^n)$	$= n x^{n-1}$
$\sqrt{x}$	$\frac{d}{dx}(\sqrt{x})$	$= \frac{1}{2\sqrt{x}}$
e^x	$\frac{d}{dx}(e^x)$	$= e^x$
a^x, $a > 0$	$\frac{d}{dx}(a^x)$	$= a^x \log a$
$\sin x$	$\frac{d}{dx}(\sin x)$	$= \cos x$
$\cos x$	$\frac{d}{dx}(\cos x)$	$= -\sin x$
$\tan x$	$\frac{d}{dx}(\tan x)$	$= \sec^2 x$
$\cot x$	$\frac{d}{dx}(\cot x)$	$= -\operatorname{cosec}^2 x$
$\operatorname{cosec} x$	$\frac{d}{dx}(\operatorname{cosec} x)$	$= -\operatorname{cosec} x \cot x$
$\sec x$	$\frac{d}{dx}(\sec x)$	$= \sec x \tan x$

SECTION A

QUESTION ONE (30 MARKS)

a) The volume of a ball (solid sphere) of radius r is given by the function $\boldsymbol{V_{(r)} = \frac{4}{3}\pi r^3}$. Find the volume of a ball of radius (3 marks)

b) Solve for x, given that $\boldsymbol{ln(2x - 1) = 2ln(x)}$ (3 marks)

c) Express $\boldsymbol{(-1 + 3i)^{-1}}$ in the form $\boldsymbol{x + iy}$. Where $\boldsymbol{x}$ and $\boldsymbol{y}$ are real numbers (3 marks)

d) Determine the exponential function $\boldsymbol{f(x) = ka^x}$ that passes through the points $\boldsymbol{(1, 6)}$ and $\boldsymbol{(2, 18)}$ (3 marks)

e) Find the **inverse** of the function $\boldsymbol{f(x) = 2x^3 + 1}$ (3marks)

f) Find $\boldsymbol{A \cdot (B \times C)}$ given that $\boldsymbol{A = -i + j + k}$, $\boldsymbol{B = i + j + k}$ and $\boldsymbol{C = i + j - k}$ (3 marks)

g) Sketch a one cycle graph of $\boldsymbol{y = 2sin(x - \pi) - 1}$ (3 marks)

h) If 5 of 20 fuses in a box are, defective and 5 of them are randomly chosen for inspection. Find the probability that two of the chosen fuses are defective (3 marks)

i) Compute the second derivative $\boldsymbol{y''}$ of the function $\boldsymbol{y = 5x^3 + 3x^2}$ (3 marks)

j) Integrate $\boldsymbol{\int e^{(5x+2)} dx}$ (3 marks)

SECTION B

QUESTION TWO (20 MARKS)

a) Solve for **x** given that $\log_2(1+\sqrt{x}) = 6$ (4 marks)

b) Solve for **x** in the equation $8^{x-2} = \sqrt{8}$ (3 marks)

c) Use chain rule to evaluate (4 marks)
$\frac{dy}{dx}[(x^2+10)^3+6]^5$

d) Obtain the first derivative y' of the function (4 marks)
$y = x^2 e^{2x^2} dx$

e) Find the slope and an equation of the normal line to the graph of (5 marks)
$f(x) = 2x + \frac{1}{\sqrt{x}}$ at a point **(1,3)**

QUESTION THREE (20 MARKS)

a) Solve for **x** in the trigonometric equation $\cos 2x + 3\sin x - 2 = 0$ (5 marks)

b) Verify the identity (4 marks)

$$\tan\theta + \cot\theta = \frac{\csc\theta}{\cos\theta}$$

c) Differentiate the function $y = x^3 \sin^2 3x$ with respect to **x** (5 marks)

d) Sketch a graph of $y = x^2 - 4x^2 + 3$ (6 marks)

QUESTION FOUR (20 MARKS)

a) Express in polar form (r, θ), the complex number $z = \sqrt{3} + i$. (4 marks)

b) Express $3e^{\frac{\pi}{4}i} = cos\frac{\pi}{4} + isin\frac{\pi}{4}$ in ordinary form $x + iy$ and sketch **plot** it on the Cartesian plane (6 marks)

c) Find $\frac{\partial z}{\partial x}$ and $\frac{\partial z}{\partial y}$ for the function $x^2 sin\, y^3 + xe^{3z} - cos\, z^2 = 3y - 6z + 8$ (10 marks)

QUESTION FIVE (20 MARKS)

a) Integrate the function using substitution method (4 marks)

$$\int_0^1 \left(\sqrt{2+x^4}\right) 4x^3 dx$$

b) Evaluate using integration by parts $\int xsinxdx$ (4 marks)

c) A line of action of force of magnitude 80 N passes through parts A (2, -2, 2) and B (4, 5, 6). Express the force in vector form (6 marks)

d) Find the probability of throwing a total of 6 points or less with three dices (6 marks)

https://trigidentities.info TRIGONOMETRIC IDENTITIES

Co-function Identities

$\sin\theta = \cos(\pi/2-\theta)$
$\sec\theta = \csc(\pi/2-\theta)$
$\tan\theta = \cot(\pi/2-\theta)$

Negative Angle Identities

$\sin(-\theta) = -\sin\theta$ $\quad$ $\csc(-\theta) = -\csc\theta$
$\cos(-\theta) = \cos\theta$ $\quad$ $\sec(-\theta) = \sec\theta$
$\tan(-\theta) = -\tan\theta$ $\quad$ $\cot(-\theta) = -\cot\theta$

Addition and Subtraction Identities

$$\sin(A+B) = \sin A\cos B + \cos A\sin B$$

$$\cos(A+B) = \cos A\cos B - \sin A\sin B$$

$$\tan(A+B) = \frac{\tan A+\tan B}{1-\tan A\tan B}$$

$$\sin(A-B) = \sin A\cos B - \cos A\sin B$$

$$\cos(A-B) = \cos A\cos B + \sin A\sin B$$

$$\tan(A-B) = \frac{\tan A-\tan B}{1+\tan A\tan B}$$

Double-Angle Identities

$$\sin 2\theta = 2\sin\theta\cos\theta$$

$$\cos 2\theta = \cos^2\theta - \sin^2\theta$$

$$= 2\cos^2\theta - 1$$

$$= 1 - 2\sin^2\theta$$

$$\tan 2\theta = \frac{2\tan\theta}{1-\tan^2\theta}$$

Product Identities

$$\sin A\cos B = \frac{1}{2}\left(\sin(A+B)+\sin(A-B)\right)$$
$$\cos A\sin B = \frac{1}{2}\left(\sin(A+B)-\sin(A-B)\right)$$
$$\cos A\cos B = \frac{1}{2}\left(\cos(A+B)+\cos(A-B)\right)$$
$$\sin A\sin B = \frac{1}{2}\left(\cos(A-B)-\cos(A+B)\right)$$

Supplement Angle Identities

$\sin(\pi-\theta) = \sin\theta$ $\quad$ $\csc(\pi-\theta) = \csc\theta$
$\cos(\pi-\theta) = -\cos\theta$ $\quad$ $\sec(\pi-\theta) = -\sec\theta$
$\tan(\pi-\theta) = -\tan\theta$ $\quad$ $\cot(\pi-\theta) = -\cot\theta$

$\sin(\pi+\theta) = -\sin\theta$ $\quad$ $\csc(\pi+\theta) = -\csc\theta$
$\cos(\pi+\theta) = -\cos\theta$ $\quad$ $\sec(\pi+\theta) = -\sec\theta$
$\tan(\pi+\theta) = \tan\theta$ $\quad$ $\cot(\pi+\theta) = \cot\theta$

Quotient Identities

$$\tan\theta = \frac{\sin\theta}{\cos\theta} \quad \cot\theta = \frac{\cos\theta}{\sin\theta} = \frac{1}{\tan\theta}$$
$$\sec\theta = \frac{1}{\cos\theta} \quad \csc\theta = \frac{1}{\sin\theta}$$

Pythagorean Identities

$$\sin^2\theta + \cos^2\theta = 1$$

$$\tan^2\theta + 1 = \sec^2\theta$$

$$\cot^2\theta + 1 = \csc^2\theta$$

Half-Angle Identities

$$\sin\frac{\theta}{2} = \pm\sqrt{\frac{1-\cos\theta}{2}}$$

$$\cos\frac{\theta}{2} = \pm\sqrt{\frac{1+\cos\theta}{2}}$$

$$\tan\frac{\theta}{2} = \pm\sqrt{\frac{1-\cos\theta}{1+\cos\theta}}$$

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Sum Identities

$$\sin A+\sin B = 2\sin\left(\frac{A+B}{2}\right)\cos\left(\frac{A-B}{2}\right)$$
$$\sin A-\sin B = 2\cos\left(\frac{A+B}{2}\right)\sin\left(\frac{A-B}{2}\right)$$
$$\cos A+\cos B = 2\cos\left(\frac{A+B}{2}\right)\cos\left(\frac{A-B}{2}\right)$$
$$\cos A-\cos B = -2\sin\left(\frac{A+B}{2}\right)\sin\left(\frac{A-B}{2}\right)$$