Enterprise Creation With Accumulated Capital By Pooling

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Abstract

Models for the formation of an enterprise created through the accumulation of resources of a group of people over time are developed. Pooled funds collected every period is locked up in a safe asset until the needed capital necessary for the creation of the enterprise are realized. Mathematical models for the management of pooled funds are developed for a general case and some specific cases. The paper demonstrates that this approach is less expensive for the enterprise as opposed to investing pooled funds in the enterprise every time funds are collected. Illustrations done via pseudo-simulated data show that enterprise formation by resource pooling is a good and viable source of funding for small and medium enterprises.

Keywords: acquisition period, crowdfunding, endorser, shadow amount, waiting period.

INTRODUCTION

Financing enterprises remain a daunting task especially for start-up enterprises as they are generally perceived as very risky by the financial and other lending institutions. Under a turbulent macroeconomic environment, sourcing finance from these institutions may not be the cheapest source of creating an enterprise especially for entrepreneurs who have limited resources to do this on their own. One way out is by resource pooling (also called crowdfunding) where a group of people who buys into the idea of the entrepreneur agree to contribute on a periodic basis in the creation of the enterprise, after which they become part owners of the enterprise. Crowdfunding is a form of fundraising whereby groups of people pool money, typically (very) small individual contributions, to support a particular goal (Ahlers et al., 2015). Five types of this mode of funding enterprises can be identified in the literature. The rewards-based, loan based, equity, charitable or donation based and real estate (Abdeldayem & Aldulaimi, 2021; Li et al., 2017). Compared to traditional funding channels, this mode of financing is more convenient for project creators to raise capital.

The challenges that confront enterprises, especially small and medium enterprises in their inability to access capital are well documented (Rajamani, et al., 2022; Madzimure, & Tau, 2021;

Gamage et al., 2020; Nkwabi & Mboya, 2019; Ahinful, 2012). Crowdfunding circumvents these financing constraints by sourcing finance directly from the public by avoiding the banks, venture capitalists, microfinance institutions and other lending institutions.

Mathematical models for management of enterprises created through the pooling of large groups of people is scanty in literature. Among the earliest work on the development of models for the management of an enterprise through resource pooling is the work of Andoh and Quaye (2013). They developed models for the management of crowdfunded enterprises under a general case and some specific scenarios. By their models, funds are invested in the enterprise each time funds are collected until the enterprise becomes operational. Investing funds in bits can raise the building cost of an enterprise as opposed to ready funds which permits the purchase of needed materials for setting up the enterprise in bulk or at once. It also makes it difficult for the entrepreneur or managers of the fund to refund amounts to members of the pool that may change their minds, become disabled or even die and can no longer contribute to the process of the enterprise creation. In addition, their model permitted contributors to earn interest on their contributions each time contributions are made prior to the acquisition period that may pose a burden on the enterprise when it becomes operational that can be minimized. The constant rate of interest employed between the time of contribution until the enterprise become operational may not reflect macroeconomic conditions at the time especially under unstable market conditions. At the recouping stage, which is a long-time horizon, a constant rate of interest is deployed will obviously not reflect macroeconomic conditions at the time. The paper is also not explicit on the proportions of shareholdings after the contributions have been paid to promoters.

The objective of the study is to develop models for the management of an enterprise created by resource pooling where the pooled funds are locked up in a safe asset until the needed funds for the creation of the enterprise are realized for a general case and some specified cases. In addition, models for the management of the enterprise when the enterprise is operational are also developed that capture macroeconomic conditions in the life of the enterprise operation.

LITERATURE REVIEW

Conceptual review

A person who buys into the idea of the entrepreneur will be called an endorser, promoter, contributor, investor, sponsor, or a participant. An enterprise created through resource pooling, also called crowding, is the contribution of a financial amount to projects, products, or business ideas by a number of investors (Bouncken, Komorek, & Kraus, 2015). Crowdfunding sometimes called crowd-capital or crowd-financing can be seen as an application of the concept of crowdsourcing, the outsourcing of problem-solving tasks to a distributed network of individuals (Gerber & Hui, 2013; Estellés-Arolas & González-LadrónDe-Guevara, 2012). Five forms of crowdfunding can be identified. The loan based crowding funding reward endorsers for their contributions but there is the possibility that an endorser may not receive anything in return. The reward-based crowdfunding permit entrepreneurs to raise funds by giving endorsers something other than capital or stock as a means of compensation. For equity-based models, investors are compensated with stakes in the enterprise (Hossain & Oparaocha, 2017; Agrawal, Catalini & Goldfarb, 2015). Donation or charitable based crowdfunding allows endorsers to donate to charitable courses without getting anything in return (Hossain & Oparaocha, 2017; Agrawal, Catalini & Goldfarb, 2015). Finally, for real estate crowdfunding, assets of numerous financial backers are pooled and used to purchase a property or assets are loaned as an advance to engineers to fund a property improvement (Abdeldayem & Aldulaimi, 2021). The models developed mimic the equity-based model, but the source of financing should not necessarily be through online platforms. In addition, the contributions made are not necessarily made once but over time. The period between when the contributions start and the enterprise get established is called the acquisition period. The pooling arrangement is said to be fully participatory if every endorser contributes every time until the enterprise is established. If at least one endorser departs or joins the pooling arrangement late, the pooling arrangement is said to be not fully participatory.

An enterprise created by endorsers can be seen as a form of partnership, a voluntary association of two or more individuals for the purpose of conducting a business for profit as co-owners.
Two basic types of partnerships can be identified: a general partnership and limited partnership. A general partnership is one in which each partner is actively involved in the management of the firm and is fully liable partnerships obligations. A limited partnership is one having at least one

general partner and one or more limited partners who are not actively engaged in partnership management and who are liable for partnership obligations only to the extent of their investment in the partnership (Black, & Skipper, 2000).

Theoretical review

Two theories can be identified to fit this study. The role congruity and stewardship theories. The role congruity theory suggests that the characteristics portrayed by the entrepreneur in a social network can motivate others to buy into the idea proposed by the entrepreneur. An individual is often perceived as part of a social group based on surface-level characteristics, such as age, race, gender, or occupation. Individuals are assumed to embody characteristics linked to a social group, regardless of whether they possess the stereotyped characteristics (Harrison et al., 1998; Anglin, et al., 2022). Role congruity theory contends that conforming to these prescribed behavioral expectations drives others’ evaluations (Eagly & Karau, 2002). In other words, people are more likely to buy in an idea so long as proponents of an idea are perceived to follow the norms of that social network. Thus, the good behaviour depicted by the entrepreneur in a social network will be a driver of pulling in endorsers to the pool. On the other hand, the stewardship theory stipulates that managers will act responsibly as stewards of the assets they control if left alone. This implies that managers, who are also stewards, will maximize the wealth of shareholders by carefully managing the resources entrusted to them. This theory has been reviewed by many researchers including Stiglitz (1975), Jensen and Meckling (1976), Mirrlees (1976), and Davis, Donaldson, and Schoorman (1997). Thus, the entrepreneur who is driving the idea of creating the enterprise will act responsibly with the contributions of the endorsers to ensure that their interest is maximized by ensuring that the enterprise come to fruition and properly managed after it becomes operational.

Empirical review

Ekpe et al. (2017) conducted a study to examine the level of awareness of crowdfunding model and its effect on entrepreneurial intentions among Nigerian university lecturers. They collected data from 217 lecturers at three universities in north, east and west regions of Nigeria and employed descriptive statistics and partial least square methods to elicit entrepreneurial interest among staff members. They discovered that overall, the model has sufficient predictive power, showing that crowdfunding (social network and website characteristics) has significant positive influence on entrepreneurial intentions among academic staff of Nigerian universities. In addition, they found that crowdfunding could lead to entrepreneurial intentions without the influence of founder's characteristics.

The issue of creation and design of webpage to show crowdfunding backers' intention to fund a project in reward crowdfunding have been discussed in Wang and Yang (2019). They adopted the elaboration likelihood model as an overarching theory to explore two questions: the kind of information that affect backers' funding intentions and how individual characteristics affect backers' funding decisions. The model was tested with survey data from China. Their findings indicate that crowdfunding project attributes (e.g., product innovativeness and perceived product quality) and creators' capabilities positively affect backers' funding intentions. Among others, their results showed that webpage visual design positively influences backers' funding intentions, but crowdfunding platform reputation does not.

In recent times, companies are adopting crowdfunding to finance more traditional products where they compete against other sellers of similar products. Miglo (2020) offers a model where two competing firms can use crowding prior to direct sales. Miglo (2020) considers a traditional framework where an entrepreneurial firm has monopoly power over its products or services.

Abdeldayem and Aldulaimi (2021) assess and evaluates crowdfunding in the Middle East and analyses the monetary requirements and the best-known ways in which middle eastern entrepreneurs raise funds. They also assessed whether crowdfunding can be viewed as an essential method for meeting the fundraising needs of entrepreneurs in the Middle East. The research sample consists of 1,910 respondents from the seven countries, namely Turkey, Egypt, Iraq, Saudi Arabia, Bahrain, Kuwait and United Arab Emirates. The respondents were investors, entrepreneurs and start-up operators. They developed a prediction model based on calculations and algorithms to forecast the success of crowdfunding projects in the Middle East and find that crowdfunding presence positively impacts fundraising success. Also crowdfunding platforms are considered an effective entrepreneurial finance tool for financing entrepreneurs in the middle east.

Mathematical models for the management of enterprises created through pooling arrangements is scanty in the literature, especially where the pooled funds are locked in safe asset before the enterprise is created. In most of works, the funds generated are not pulled over time which is a point of departure from this study. The closest is the work of Andoh and Quaye (2013) with deficiencies alluded to earlier for which this paper rectifies.

MATERIALS AND METHODS

Development of General models

Suppose a group of people κ decide to contribute to a fund to create an enterprise at a price T_f at the end of time N, the acquisition period. It should be noted that T_f incorporate compensation for providing funds by the endorsers. Let the amount contributed to the fund at time $N' < N$, T_f', be sufficient to create the enterprise. Suppose at time j, $j = 1, \ldots, N'$ each participant i, $i = 1, \ldots, \kappa$ contribute an amount s_{fij}, $i = 1, \ldots, \kappa$; $j = 1, \ldots, N'$. Then $\sum_{i=1}^{\kappa} s_{fi1}$ will be collected at time 1 and suppose that this amount grows to $S_{F1N'}$ at time N'. At time 2, $\sum_{i=1}^{\kappa} s_{fi2}$ will be collected which would have accumulated to $S_{F2N'}$ at time 2. Continuing in this fashion, at time N', $\sum_{i=1}^{\kappa} s_{fiN'}$ which will grow to $S_{FN'N'}$. At N' the value of the fund is $\sum_{j=1}^{N'} S_{FjN'}$ which must coincide with T_f', the amount needed to create the enterprise. Between the period N' and N, endorsers must be compensated for the waiting period, the period between when the funds are available and when the enterprise is created. Let r be the continuous rate of interest. Then the value of the contributions at time N is $\sum_{j=1}^{N'} S_{FjN'}(1+r)^n$ and this must coincide with T_f, the cost of the enterprise at time N from the perspective of the endorsers. Thus

$$\sum_{j=1}^{N'} S_{FjN'}(1+r)^n = T_f \qquad (1)$$

where $n = N - N'$ from which we see that

$$N = N' + \log_{(1+r)}\left(\frac{T_f}{\sum_{j=1}^{N'} S_{FjN'}}\right)$$

Remark 1: This formulation makes it easier for the managers of the fund to refund the contributions (though strongly discouraged) of members who might have changed their minds by time N'.

Beyond this time, funds would have been deployed and endorsers are not ordinarily permitted to change their minds about withdrawal. In addition, it reduces the burden on the enterprise when the enterprise become operational: the enterprise only pay interest on contributions only for the period n, the waiting period for the enterprise to be established as opposed to paying interest each time contributions are made by endorsers.

In the general case, no restriction is placed on the time of entry to the pool until time N'. Beyond this time, the needed capital would have been realized and no new entrant would be accommodated. The proportion of each endorser i stake at time j is given by $\frac{s^{ij}}{S_{FjN'}}, i = 1, \ . \ . \ . \ , \kappa$

and proportion of each endorser i stake at time N' is given by

$$\frac{\sum_{j=1}^{N'} S_{ij}}{\sum_{j=1}^{N'} S_{FjN'}}, i = 1, \ . \ . \ . \ , \kappa$$

which remain the same under constant rate of interest by time N.

Proposition 1: For all $r \in (0,1]$ and $N, N' \in \mathbb{N}$ with $N' < N$,

$$\sum_{j=1}^{N-N'}(1+r)^j < \sum_{j=1}^{N}(1+r)^{N-j} \tag{2}$$

Proof: Let $S_1 = \sum_{j=1}^{N-N'}(1+r)^j$ and $S_2 = \sum_{j=1}^{N}(1+r)^{N-j}$ it suffices to show that $S_2 - S_1 > 0$. First observe that

$$S_2 = \sum_{j=1}^{N}(1+r)^{N-j} = \sum_{j=1}^{N}(1+r)^{j-1} = \sum_{j=1}^{N-N'}(1+r)^{j-1} + \sum_{j=N-N'+1}^{N}(1+r)^{j-1}$$

$$= \sum_{j=0}^{N-N'-1}(1+r)^j + [(1+r)^{N-N'+1} + \sum_{j=N-N'+1}^{N}(1+r)^{j-1}]$$

$$= 1 + \sum_{j=1}^{N-N'-1}(1+r)^j + (1+r)^{N-N'} + \sum_{j=N-N'}^{N}(1+r)^{j-1}$$

Now

$$S_1 = \sum_{j=1}^{N-N'} (1+r)^j = \sum_{j=1}^{N-N'-1} (1+r)^j + (1+r)^{N-N'}$$

and so $S^2 - S_1 = 1 + \sum_{j=N-N'}^{N} (1+r)^{j-1} > 0$ for all $r \in (0,1]$.

Note that LHS of (2) is the interest earned on the contributions up to when the enterprise becomes operational (i.e., the amount the endorsers are demanding on the enterprise for providing the capital for setting the enterprise up). On the other hand, the RHS of (2) of Proposition 1 is the amount endorsers are demanding on the enterprise each time the contributions are made and are invested until the enterprise becomes operational. Proposition 1 essentially emphasizes the point that it is less expensive for the enterprise to get the needed funds ready before setting the enterprise up. An enterprise can be regarded as a living organism providing life to other stakeholders whose life hinges on its survival. Consequently, decisions or efforts that go to reducing its burden is good not only for the survival of the enterprise but the endorsers whose sweat created the enterprise including other stakeholders whose lives depend on it.

Remark 2: It is expected that n in equation (1) will be small (typically not exceeding a year) when the needed funds for establishing the enterprise are realized. In such as case, keeping a constant rate r is reasonable. On the other hand, if n is large (exceeds a year), it may be necessary to adjust the rate of interest to account for the prevailing macroeconomic conditions at the time.

Let r_m, be the continuous rate of interest at time $m = 1, \ . \ . \ . \ , n.$ Then

$$\sum_{j=1}^{N'} S_{F_{jN'}} \left[\prod_{m=1}^{n} (1 + r_m)^m\right] = T_f \quad (3)$$

which can be written

$$\sum_{m=1}^{n} m \log(1 + r_m) = \log \left(\frac{T_f}{\sum_{j=1}^{N'} S_{F_{jN'}}}\right).$$

Remark 3: In the case where interest rate changes every period, the same argument as in Proposition 1 with r replaced with r_j shows that it is less expensive to the enterprise to have the funds ready before the creation of the enterprise.

One of the primary concerns for every endorser is how long it will take for the enterprise to be established, so they can recoup of their investments. To determine the waiting period in (3), the equation must be solved numerically. See for example, Brandimarte (2002), pps 111-117.

Claim 1: For all $n \in \mathbb{N}, r \in (0,1], r\prod_{j=1}^{n} j + 1 \leq \prod_{j=1}^{n}(jr+1)$.

Proof: The proof will be accomplished by induction. Let $P(n)$ be

$$r\prod_{j=1}^{n} j + 1 \leq \prod_{j=1}^{n}(jr+1).$$

$P(1): r\prod_{j=1}^{1} j + 1 \leq \prod_{j=1}^{1}(jr+1)$. which is true as $1 + r \leq (1 + r)$ for all $0 < r \leq 1$.

$P(k): r\prod_{j=1}^{k} j + 1 \leq \prod_{j=1}^{k}(jr+1)$ from which it follows that $r\prod_{j=1}^{k} j < \prod_{j=1}^{k}(jr+1)$.

$P(k+1): r\prod_{j=1}^{k+1} j + 1 \leq \prod_{j=1}^{k+1}(jr+1)$. Now it suffices to proof that $P(k)$ imply $P(k+1)$ and conclude that $P(n)$ is true for all $n \in \mathbb{N}$. From the LHS of $P(k+1)$,

$$r\prod_{j=1}^{k+1} j + 1 = r(1)(2)\cdots k(k+1) + 1$$

$$= [r\prod_{j=1}^{k} j]\,(k+1) + 1$$

$$< [r\prod_{j=1}^{k}(jr+1)]\,(k+1) + 1$$

By the implied $P(k)$. Thus

$$r\prod_{j=1}^{k+1} j + 1 \leq \prod_{j=1}^{k}(jr+1)\,(k+1)r + 1$$

$$\leq \prod_{j=1}^{k+1}(jr+1)$$

This is $P(k+1)$! $P(1)$ is true and $P(k)$ implies $P(k+1)$. Therefore, by induction $P(n)$ is true for all $n \in \mathbb{N}$.

Proposition 2: For all $0 < r \leq 1$,

$$\sum_{j=1}^{N'} S_{F_jN'} < \frac{1}{nr+1}\sum_{j=1}^{N'} S_{F_jN'}\left[\prod_{m=1}^{n}(1+r)^m\right] < \sum_{j=1}^{N'} S_{F_jN'}\left[\prod_{m=1}^{n}(1+r)^m\right] \quad (4)$$

where $n = N - N'$.

Proof: Observe that from the Andoh's inequality, $nr \leq (1+r)^n - 1$, that we can write

$$jr + 1 \leq (1+r)^j$$

for all $j \in \mathbb{N}$. Hence $\prod_{j=1}^{n}(jr+1) \leq \prod_{j=1}^{n}(1+r)^j$. It follows by Claim 1 that $r\prod_{j=1}^{n} j + 1 \leq \prod_{j=1}^{n}(1+r)^j$. Thus,

$$rn! + 1 \leq \prod_{j=1}^{n}(1+r)^j$$

Therefore,

$$1 \leq \frac{1}{rn!+1}\prod_{j=1}^{n}(1+r)^j < \frac{1}{rn+1}\prod_{j=1}^{n}(1+r)^j \quad (5)$$

Because $0 < r \leq 1$ we can write $1 < rn + 1 \leq (n+1)$. Therefore

$$1 > \frac{1}{rn+1} \geq \frac{1}{n+1}.$$

From the inequality (5), we can write

$$1 < \frac{1}{rn+1}\prod_{j=1}^{n}(1+r)^j < \prod_{j=1}^{n}(1+r)^j$$

as $\frac{1}{rn+1} < 1$ and multiplying this last inequality through by $\sum_{j=1}^{N'} S_{F_jN'}$ gives

$$\sum_{j=1}^{N'} S_{F_jN'} < \frac{1}{nr+1}\sum_{j=1}^{N'} S_{F_jN'}\left[\prod_{m=1}^{n}(1+r)^m\right] < \sum_{j=1}^{N'} S_{F_jN'}\left[\prod_{m=1}^{n}(1+r)^m\right]$$

The quantity, $\frac{1}{nr+1}\sum_{j=1}^{N'} S_{FjN'}\left[\prod_{m=1}^{n}(1+r)^m\right]$, in equation (4) do not offer full compounded amount in the waiting period. It offers a smaller amount accounting for the difficulty in creating and enterprise. Compare this result with Andoh and Quaye (2013), pp 51.

Now the expression

$$\frac{1}{nr+1}\sum_{j=1}^{N'} S_{FjN'}\left[\prod_{m=1}^{n}(1+r)^m\right] - \sum_{j=1}^{N'} S_{FjN'} = \frac{1}{nr+1}T_f - T_f'$$

give the shadow amount, the amount endorsers are demanding within the waiting period.

Note that delays in establishing the enterprise after time N' reduces the shadow amount. Consequently, this could assist endorsers mount some pressure on the entrepreneur or the managers of the fund to ensure that any unnecessary delays on the part of the entrepreneur or managers of the fund is curtailed.

Remark 4: If interest rate changes every period,

$$\sum_{j=1}^{N'} S_{FjN'} < \frac{1}{nr_m+1}\sum_{j=1}^{N'} S_{FjN'}\left[\prod_{m=1}^{n}(1+r_m)^m\right] < \sum_{j=1}^{N'} S_{FjN'}\left[\prod_{m=1}^{n}(1+r_m)^m\right].$$

Note: In computing the shadow amount, $\frac{1}{nr_m+1}T_f - T_f'$, in this case the most representative value

(the mean, median) of r_m can be used. Alternatively, it can be computed for all values of r_m and the magnitude of the resulting vector used.

Proposition 3: For all $r_j \in (0,1]$, $T_f' < T_f \le 2^{\frac{1}{2}n(n+1)}T_f'$.

Proof: Because $0 < r \le 1$, we can write $1 < (1+r)^j \le 2^j, j = 1, \ldots, n$. Therefore,

$$\prod_{j=1}^{n} 1 < \prod_{j=1}^{n}(1+r)^j \le \prod_{j=1}^{n} 2^j$$

which can be written as $1 < \prod_{j=1}^{n}(1+r)^j \leq 2^{\frac{1}{2}n(n+1)}$. Using this last inequality we can write

$$\underbrace{\sum_{j=1}^{N'} S_{FjN'}}_{T_f'} < \underbrace{\sum_{j=1}^{N'} S_{FjN'}\left[\prod_{j=1}^{n}(1+r)^j\right]}_{T_f} \leq 2^{\frac{1}{2}n(n+1)} \underbrace{\sum_{j=1}^{N'} S_{FjN'}}_{T_f'}$$

and so, the result follows.

This proposition places a cap on how much endorsers can demand on their investment. It is to check exploitation of the enterprise by promoters or endorsers. An enterprise can be thought of as a living organism on whose life other stakeholders depend. Therefore, any unnecessary demand by promoters can stifle the enterprise and render other stakeholders jobless. This proposition is there to check this from happening.

Proposition 4: For all $r \in (0,1]$, $n \in [(-1+\sqrt{1+4x})/2\,, \infty)$ where $x = \frac{2\log(\frac{T_f}{T_f'})}{\log 2}$

Proof: Observe from the upper bound of Proposition 3 that $T_f \leq 2^{\frac{1}{2}n(n+1)} T_f'$. Extracting n from this inequality, we see that $n^2 + n - x \geq 0$, where $x = \frac{2\log(\frac{T_f}{T_f'})}{\log 2}$. Solving this inequality yields

$$n \leq \frac{-1-\sqrt{1+4x}}{2} \text{ or } n \geq \frac{-1+\sqrt{1+4x}}{2}.$$

Only the latter inequality is admissible and so the result follows.

Proposition 4 provides the minimum waiting time on the entrepreneur or managers of the fund to get the enterprise established. Beyond this time point, endorsers should be expectant of coming to fruition of enterprise. It serves as a check on the entrepreneur or managers of the funds responsible for ensuring the enterprise materializes within a minimum time. The inequality also indicate that the enterprise may not come to fruition and so monitoring by endorsers may be necessary.

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Some Special cases

Proposition 5: Let $r \in (0,1]$, $n \in \mathbb{N}$. If every contribution grows to an amount S_F for all $j = 1, \ldots, N'$, then the total amount at the end of the acquisition period is $N'S_F(1+r)^n$ and investors have to wait for $log_{(1+r)}\left(\frac{T_f}{T_{f'}}\right)$ for the enterprise to be established.

Proof: Observe that $\sum_{j=1}^{N'} S_{F_{jN'}} = \sum_{j=1}^{N'} S_F = N'S_F$ and so the total amount at the end of the acquisition period is $N'S_F(1+r)^n$. Note that $T_f = N'S_F(1+r)^n$ and $T_f{}' = N'S_F$. Therefore, $T_f = T_f{}'(1+r)^n$ and taking logs of both sides yields the required result.

Proposition 6: Let $S_{F_{jN'}} \in U[\alpha, \beta]$, $j = 1, \ldots, N'$. Then as $N' \to \infty$, $T_f' = N'\left(\frac{\alpha+\beta}{2}\right)$ and $n \in [log_2(1+r) - 1, \infty)$, $r \in (0,1]$.

Proof: The proof follows as in Andoh and Quaye (2013). For the second part, note that as

$$T_{f'} = N'\left(\frac{\alpha+\beta}{2}\right), \quad T_f = N'\left(\frac{\alpha+\beta}{2}\right)(1+r)^n$$

and applying Proposition 3, yields

$$(1+r)^n \leq 2^{\frac{1}{2}n(n+1)}$$

From which we see that $n \geq 2log_2(1+r) - 1$.

Risk management matters

Leaving the arrangement within the waiting period is ordinarily not allowed. There might be special cases where this might arise. If an endorser becomes temporally or totally disabled or contract a fatal illness for which large medical expense will be needed. An endorser might even die for which the next of kin may need funds to cater for the needs of growing children. In all these cases, there will be the need to refund even within the waiting period and for that matter the need to manage this risk by the remaining endorsers.

One way to manage the risk is for the remaining endorsers to split the cost of the refund in proportion to their stake at time N'. Also, managers of the enterprise can look for a third party who may purchase the interest of the endorser.

Key personnel insurance can also be purchased by the endorsers at time N'. Let P_{im} be the premium paid at time $m = 1, \ . \ . \ . \ , n$ by each endorser $i = 1, \ . \ . \ . \ , \kappa$. Then the total premium

paid by all endorsers at time N to the insurer is given by

$$\sum_{i=1}^{\kappa} \sum_{m=1}^{n} P_{im}$$

Accounting for the time value of premium paid from the endorsers' perspective,

$$\sum_{i=1}^{\kappa} \sum_{m=1}^{n} P_{im} (1 + r)^n$$

would have been injected by the endorsers. Adding this to equation (1) give

$$\sum_{j=1}^{N'} S_{F_{jN'}} (1 + r)^n + \sum_{i=1}^{\kappa} \sum_{m=1}^{n} P_{im} (1 + r)^n$$

which is the cost of the enterprise at the acquisition period N. If the interest rate varies within the period, the cost of the enterprise at the acquisition period is

$$\sum_{j=1}^{N'} S_{F_{jN'}} [\prod_{m=1}^{n}(1 + r_m)^m] + \sum_{i=1}^{\kappa} \sum_{m=1}^{n} P_{im} [\prod_{m=1}^{n}(1 + r_m)^m]$$

Alternatively, the endorsers can agree to create a premium account at time $j = 1, \ldots, N'$ to cater for the cost of insurance. Let each endorser contribute an amount p_{ij}, $i = 1, \ . \ . \ . \ , \kappa; j = 1, \ . \ . \ . \ , N'$ to the premium account. Then $\sum_{i=1}^{\kappa} p_{i1}$ will be collected at time 1 and let this amount grow to that grow to $\tilde{P}_{1N'}$ at time N'. Also, $\sum_{i=1}^{\kappa} p_{i2}$ will have grown to $\tilde{P}_{2N'}$. Continuing in this manner, $\sum_{j=1}^{N'} \tilde{P}_{jN'}$ be available in the premium account at time N'. The total cost of the enterprise from the endorser's perspective will be

$$\sum_{j=1}^{N'} S_{F_{jN'}}(1+r)_n + \sum_{j=1}^{N'} P_{jN'}(1+r)_n$$

under constant rate of interest or

$$\sum_{j=1}^{N'} S_{F_{jN'}}\left[\prod_{m=1}^{n}(1+r_m)^m\right] + \sum_{j=1}^{N'} P_{jN'}\left[\prod_{m=1}^{n}(1+r_m)^m\right]$$

if interest rate varies.

Remark 5: Because funds are saved in a premium account it makes it easy to refund the contributions of endorsers who may have changed their minds before time N'.

Recouping contributions of promoters or endorsers

In this section, mathematical models for the management of the enterprise that have been created to ensure sustainability and payback of the promoters will be developed. Let P_{fi} be the profit function of the enterprise operations for period $i, i = 1, \ . \ . \ . \ . \ , M, M \in \mathbb{N}$. Then

$$P_{fi} = t_{ri} - t_{ci}$$

where t_{ri} and t_{ci} are the total revenue and total cost of operations for each period i. P_{fi} can be written as

$$P_{fi} = t_{ri} - [t_f + v_{ci} + T_f\omega_i(1+r_i)^i], i = 1, \ . \ . \ . \ . \ , M$$

(5) where $\mathbf{0 < \omega_i \leq 1}$, $\sum^{n}_{i=1} \boldsymbol{\omega_i} = \mathbf{1}$, $\boldsymbol{t_f}$ and $\boldsymbol{v_c}$ are respectively the fixed and variable cost of operations for each period. $\boldsymbol{r_i}$ is the interest rate for each period. Because $\boldsymbol{M}$ is typically large, it is unrealistic to keep the interest rate as constant. This contrasts with Andoh and Quaye (2013) who permitted the interest rate to be constant within the payback period. The expression $\boldsymbol{T_f\omega_i(1 + r_i)^i}$ is the strain on the enterprise for period $\boldsymbol{i}$, the amount that belongs to the

endorsers for each period. For sustainability of the operations of the enterprise,

$$t_{ri} > [t_f + v_{ci} + T_f \omega_i (1 + r_i)_i]$$

and if t_{ri} is obtained by selling a certain number of products n_p at a price s_p then

$$n_p > \frac{t_f + v_{ci} + T_f \omega_i (1 + r_i)_i}{s_p}$$

Here, the enterprise is assumed to be in the production of goods and/or services where the price is fixed by the market. On the other hand, if n_p is held constant, then the interest is to determine the price s_p for sustainability. In this case,

RESULTS AND DISCUSSION

The interest rates used for the analysis are the 91-day treasury rates obtained from Bank of Ghana for the period January 2017 to December 2021 totaling 60 data points. 48 of these data points (from January 2017 to December 2020) will be used to estimate the values of $S_{F_{jN'}}, j =$

$1, \ . \ . \ . \ ,48$ at time $N' = 48$. The remaining 12 data points will be used to estimate the cost to the enterprise by the endorsers for providing the required funding. The 91-day treasury rates for 2022 was excluded from the analysis due to the domestic debt exchange programme the Government of Ghana was instituting on domestic bondholders resulting in high treasury rates especially in the last half of 2022. Investors were shying away from investment in bonds and given that the government was struggling to obtain money, it turned its attention to the treasury market forcing the rates up. Thus, the rates reported are not the norm and will swell up the cost of the enterprise.

Suppose that $T_f{}' = 3{,}000{,}000$, $\kappa = 50$ people, $N' = 48$ months, $N = 60$ months so that $n = 12$

months. Let $s_{fij} \in |N(1000, 500^2)|, i = 1, 2, \ . \ . \ . \ ,50; j = 1, \ . \ . \ . \ ,48$ of size 50 by 48 be generated and assume that these cumulative monthly contributions are invested in 91-day treasury bills for 48 months. Investment in treasury bills are risk free and so endorsers are certain of the funds and its interest. From the simulated data s_{fij} lies in [1.12,2789.20] and assume that these amounts are in Ghana cedis, the currency of the Republic of Ghana. The lower plot of Figure 1 (the solid line) shows the cumulative monthly contributions whereas the upper plot (dashed line) shows the growth amounts $S_{FjN'}$, $j = 1, \ . \ . \ . \ ,48$ at the end of 48-month period.

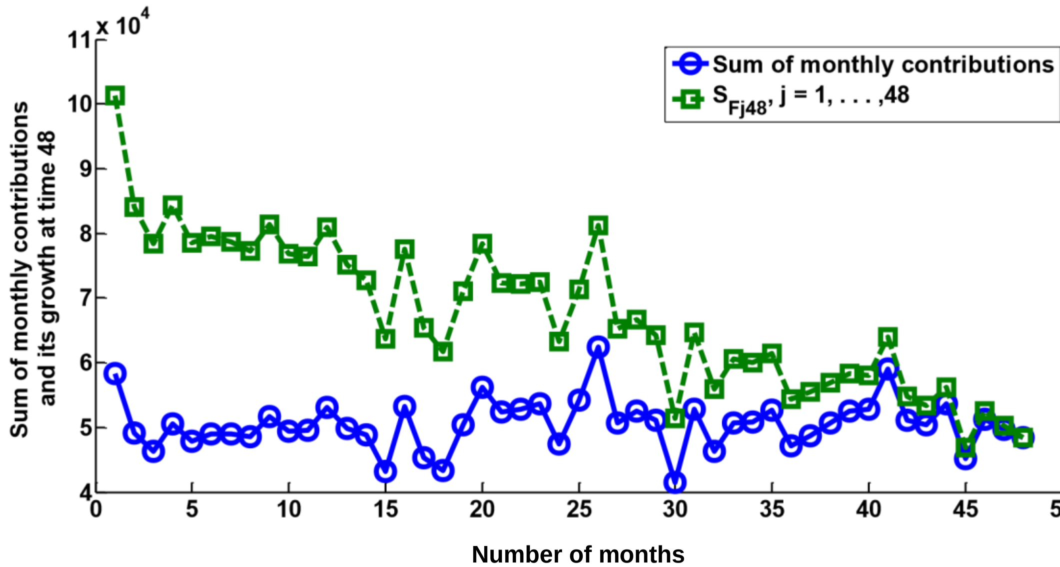

Figure 1: Contributions (solid circled line) versus growth of contributions at month 48 (dashed squared line)

The total contributions by the 50 endorsers at the end of the 48 months is $\sum_{i=1}^{50}\sum_{j=1}^{48} s_{fij} = 2.404 \times 10^6$ will have grown to the to an amount $\sum_{j=1}^{48} S_{Fj48} = 3.215 \times 10^6 = T_f{'}$. The difference 8.11×10^5 is the savings to the enterprise. Now from equation (3), and using the remaining 12 data points give

$$\sum_{j=1}^{48} S_{FjN'} \left[\prod_{m=1}^{12}(1 + r_m)^m\right] = 4.1157 \times 10^7 = T_f.$$

The cost to the enterprise prior to commencement of operations is 4.1157×10^7 which would have been at least $(4.1157 \times 10^7 + 8.11 \times 10^5)$ should the investment in the enterprise been done each time funds are collected.

Now let us investigate the growth in individual contributions made over the 48 months for the 50 endorsers at the end of 48 and 60 months. Figure 2 demonstrates the power of resource pooling. At time 48 when no more contributions will be made, each endorser investment will

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have growth to 1.34 times their initial investment. Within 12 months when no contributions were made and the amount invested together, the individual initial investment would have growth nearly to 17-fold. This shows the power of resource pooling of a large group of people.

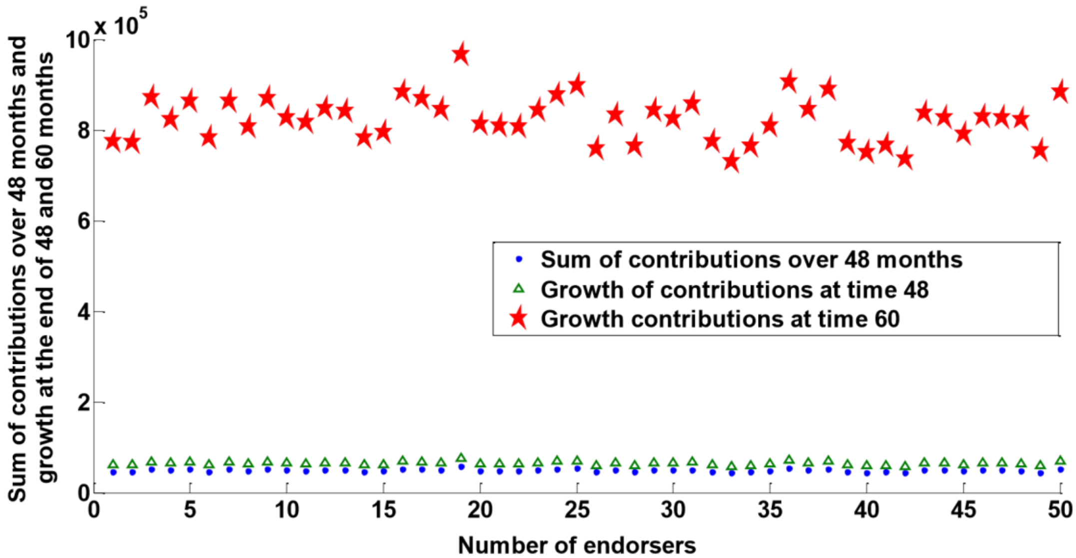

Figure 2: Sum of contributions over 48 months (□), growth of contributions at time 48 (△) and growth of contribution at time 60 (*).

How will increasing the number of endorsers have on the fund accumulated at time 48 and at time 60? Do the increasing numbers of endorsers affect the quantum of individual investments at time 48 and at time 60? As can be seen from the table 2, both the values of $T_f{}'$ and T_f increases with increasing number of endorsers, the value of T_f rises much faster. Observe also that the number of times the individual investment grows at both times 48 and 60 are largely unaffected with increasing number of endorsers.

Table 2: Total contributions of endorsers, value of $T_f{}'$, value of T_f, the number of times individual grows at time 48 and the number of times individual investment grow at time 60.

Number of endorsers	Total contributions of endorsers at time 48 ($\times \mathbf{10^7}$)	Value of $\boldsymbol{T_f}'$ at time 48 ($\times \mathbf{10^7}$)	Value of $\boldsymbol{T_f}$ at time 60 ($\times \mathbf{10^8}$)	Number of times individual investment grow at time 48	Number of times individual investment grow at time 60
25	0.1219	0.1623	0.2064	1.3306	16.9285
50	0.2404	0.3215	0.4116	1.3373	17.0130
75	0.3654	0.4879	0.6207	1.3353	16.9879
100	0.4817	0.6428	0.8178	1.3345	16.9775
125	0.6069	0.8093	1.0296	1.3334	16.9637
150	0.7194	0.9614	1.2231	1.3364	17.0020
175	0.8468	1.1321	1.4402	1.3369	17.0080
200	0.9644	1.2891	1.6400	1.3366	17.0049
225	1.0899	1.4567	1.8532	1.3365	17.0030
250	1.2067	1.6096	2.0477	1.3338	16.9689
275	1.3285	1.7727	2.2552	1.3343	16.9754
300	1.4522	1.9396	2.4675	1.3356	16.9921
325	1.5796	2.1110	2.6856	1.3364	17.0021
350	1.6883	2.2580	2.8727	1.3374	17.0150
375	1.8120	2.4192	3.0777	1.3350	16.9845
400	1.9426	2.5947	3.3010	1.3357	16.9926
425	2.0694	2.7615	3.5132	1.3345	16.9771
450	2.1752	2.9060	3.6971	1.3360	16.9967
475	2.2951	3.0648	3.8990	1.3353	16.9883
500	2.4240	3.2369	4.1180	1.3353	16.9882

Figure 3 shows the total contributions (solid line), the growths at times 48 (dashed line) and 60 (dotted line) and the shadow amount (dash-dot line). The wide variation between the growth at time 48 and 60 can clearly be seen from the plot.

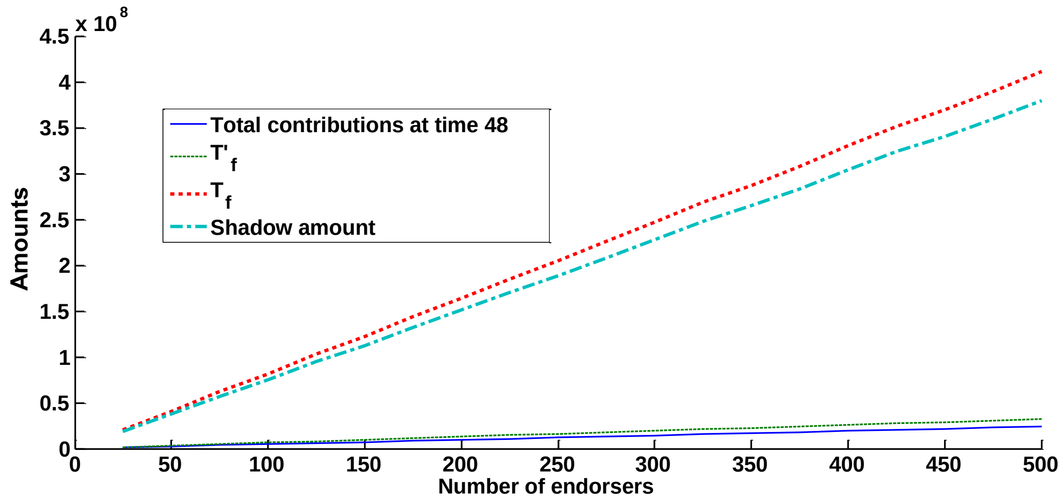

Figure 3: Total contributions of endorsers (solid line), value of $T_f{}'$ (dashed line), value of T_f (dotted line) and the shadow amount (dash-dot line).

Now suppose that $T_f{}'$ is sufficient to set up a Science High School with operating expenses depicted in table 3. The viability of such a venture using the models discussed in section 4 will be assessed using a hypothetical interest rate uniformly distributed in the interval [0.05, 0.07]. Such an enterprise is generally set up to live in perpetuity and so M is chosen as 100.

Table 3: Cost structure of setting up the school[1].

Cost type	Components	Amount ($)
Fixed cost (t_f)	Teaching staff	150,000
	Administrative staff	75,000
	Security staff	9,000
	Social Security	28,125
	Insurance	60,000
	Totals	322,125
Variable cost (v_c)		

[1] At the time of the computations 1USD $\cong GHS12$.

Specific expenses	Admission expenses	4000
	Information pack	10000
	Feeding cost	50400
General expenses	Utilities	25000
	Vehicle running expenses	30,000
	Maintenance/breakages	4500
	Miscellaneous	25000
	Subtotal	84500
T_f	Value of endorsers contribution at the end of acquisition period	$\frac{4.1157 \times 10^7}{12}$
	Totals	3.43×10^6

Typically, the fixed cost is the cost of paying teachers, administrative staff, security personnel, social security payments for staff and insurance cost. Workers' salaries typically do not change within a year but may alter from year to year depending on the macroeconomic conditions. The variable cost will be separated into two parts: specific expenses and general expenses. The specific expenses are those variable costs that vary directly as the number of students enrolled in the school. These include the cost of feeding students, admission expenses and information pack expenses (cost of printing brochures, flyers, radio ads, etc). The general expenses on the other hand, are those variable costs incurred in the running of the school. These costs include utilities, maintenance costs and vehicle running costs. Other general expenses such as refreshment for meetings, stationery, generator running cost, cleaning cost, subscriptions, and teaching aids will be bundled together as miscellaneous expenses.

Denote the feeding expenses, admission expenses and information pack expenses for year i respectively by FE_i, AE_i and IPE_i. They are given by

$$FE_i = k_1 n_i,\ AE_i = k_2 n_i \text{ and } IPE_i = k_3 n_i$$

where k_1, k_2 and k_3 are constants representing the cost per student for feeding, admissions, and information pack expenses and n_i is the number of students for year i. Thus, the variable cost become

$$v_{ci} = v_{csei} + v_{cgei}$$

where $v_c{}^{se}{}_i$ is the specific expenses component of the variable cost and $v_c{}^{ge}{}_i$ is the general expenses component of the variable cost.

Thus, equation (5) becomes

$$P_{fi} = t_{ri} - [t_f + v_c{}^{se}{}_i + v_c{}^{ge}{}_i + T_f\omega_i(1 + r_i)^i], i = 1, \ . \ . \ . \ ,100 \quad (6)$$

P_{fi} will be investigated for moderate student numbers 120, 240, 360, 360, …, 360 enrolled using student fees of 3600 per year. It should be noted that

$$v_{ci} = v_c{}^{se}{}_i + v_c{}^{ge}{}_i = \frac{100}{3} n_i + \frac{250}{3} n_i + 420n_i + 84500 \quad (7)$$

where the constants k_1, k_2 and k_3 were obtained using the year 1 student numbers depicted on table A1 at the appendix and the specific and general expenses values indicated in table 3.

Inserting (7) into (6) and the values in table 3, gives

$$P_{fi} = 3600n_i - [322125 + \frac{100}{3} n_i + \frac{250}{3} n_i + 420n_i + 84500 + 3.43 \times 10^6 \times \omega_i(1 + r_i)^i, i = 1, \ . \ . \ . \ ,100]$$

which reduces to

$$P_{fi} = 3063.33n_i - 406625 - 3.43 \times 10^6 \times \omega_i(1 + r_i)^i, i = 1, \ . \ . \ . \ ,100.$$

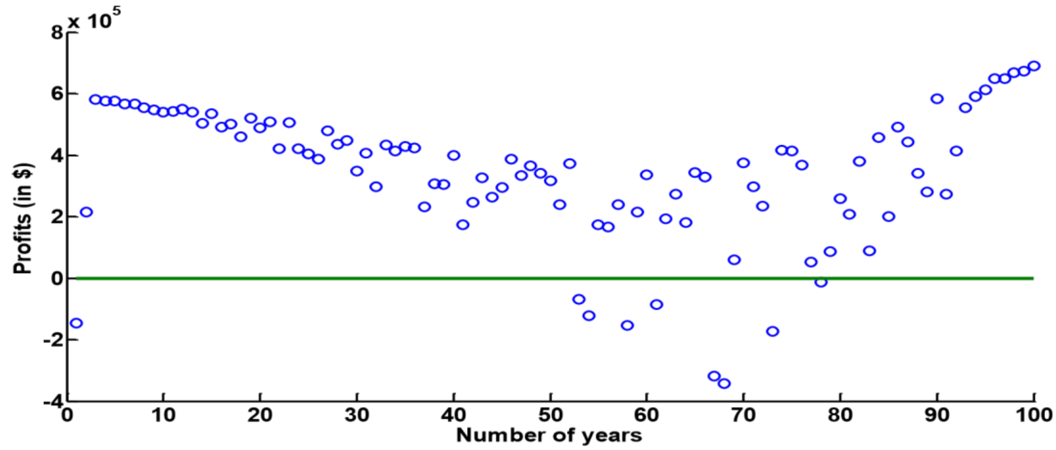

Figure 4: Profits (°) of the school over 100 years and the break-even line (solid line).

Within the 100-year period, the school will make losses in 9 of the years. Generally, it appears from the plot that it is a good venture for endorsers as they will be making significant gains. Aside the strain they receive each year (see upper plot of figure 5), they are also entitled to a portion of the yearly profit. The weights applied to the T_f is shown in the lower plot of figure 4.

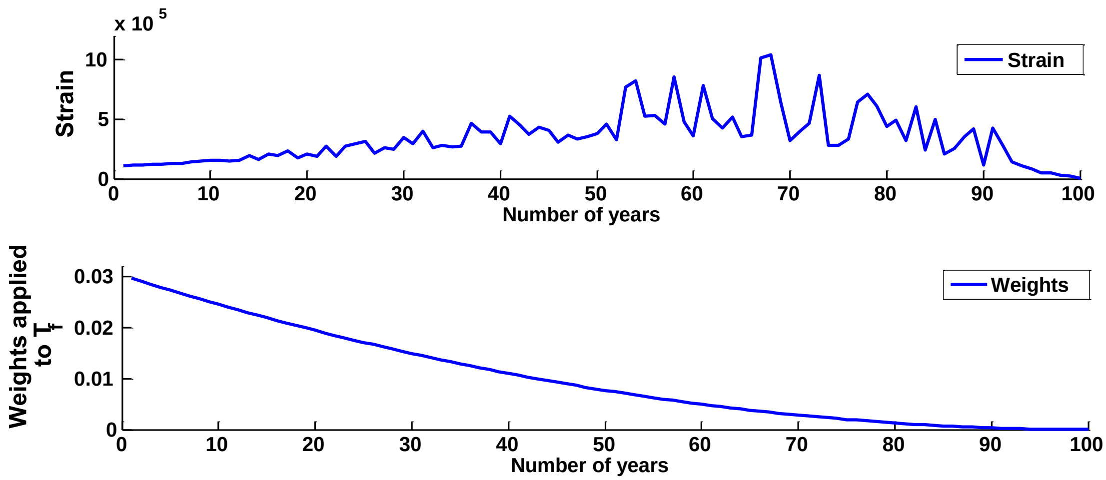

Figure 5: Upper plot: periodic payment to endorsers (strain).

Lower plot: weights applied to T_f.

How should the losses be minimized? One way is the application of Proposition 2. From RHS of Proposition 2 (inequality (4)), and using the remaining 12 data points with $r_m = 0.06$ give

$$\frac{2}{nr_m+1}\sum_{j=1}^{48}\left[\prod_{m=1}^{12}(1+r_m)^m\right]=\frac{1}{12r_m+1}$$

$$S_{FjN'}T_f = 2.3928\times 10^7$$

Inserting this value in place of 4.1157×10^7 in table 3 results in the profitability indicated in the figure 6 below.

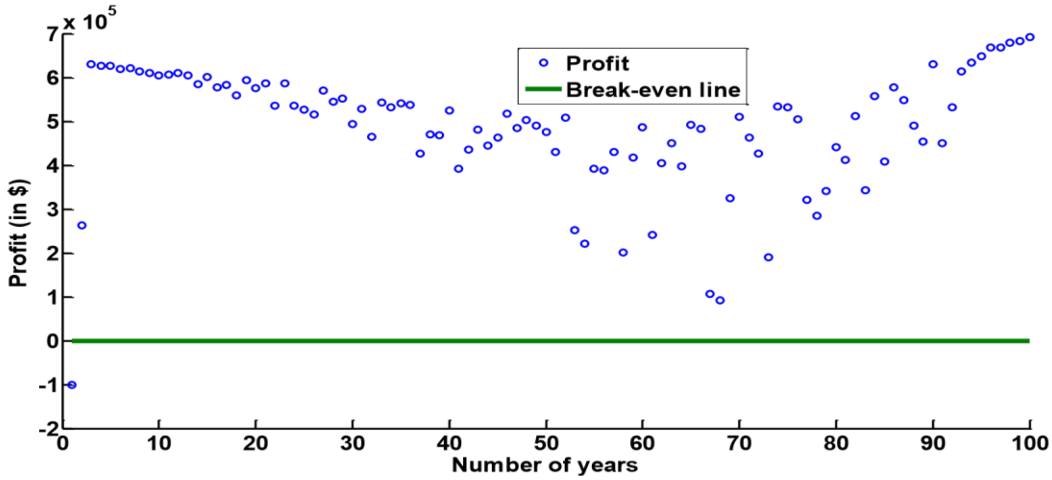

Figure 6: Profits of the school over 100 years with break-even line. The T_f was computed using proposition 2.

It can be observed from figure 6 that except for the first year where the enterprise incurred a loss, there was profit for all the years.

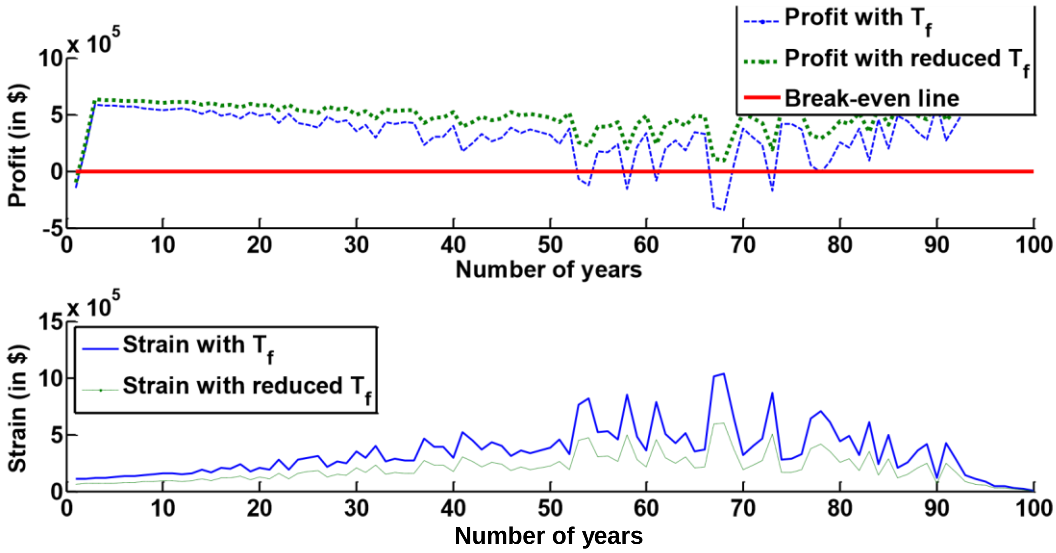

Figure 7: Upper plot: Profit with T_f (dashed line) and with reduced T_f (dotted line) and the break-even line (solid line). Lower plot: Strain with both T_f (solid line) and reduced T_f (dashed line).

Clearly from figure 7, there is reduction in the amount due endorsers with the application of Proposition 2. On the other hand, the profits are higher with Proposition 2 indicating a transfer of wealth from the strain to the profits. This allows the wealth to be spread out to all stakeholders intead of merely concentrating on the endorsers.

CONCLUSION AND RECOMMENDATIONS

It has been demonstrated mathematically and through pseudo-simulated data that investing a stream of contributions from a large group of people over time in a safe asset until the needed funds required to set up an enterprise is less expensive to the enterprise as opposed to investing funds in the enterprise each time the funds are collected. Through the pseudo-simulated data, the power of resource pooling over time to fund the enterprises can be seen. The possibility of loss is minimized when endorsers are not offered the full compounded amount during the waiting period. For contributions made over the same time, the number of times individual contributions grow at the end of the acquisition period remains approximately the same irrespective of the number of people in the pool.

The models are good decision tools for the management of enterprises created through resource pooling. The models permit endorsers to determine their stake in the enterprise during the acquisition period and after the enterprise has been created and is operational. It also provides endorsers with a sense of the time frame for enterprise to be operational for recouping of funds to start. This knowledge by endorsers puts managers of the fund on the edge to ensure the creation of the enterprise comes to fruition.

Aside individuals who can team-up to create enterprises for the growing number of university graduates, organizations, companies can team up to create enterprises free from borrowing from lending institutions. Afterall, the funds available to most of these lending institutions emanate from individuals, enterprises, and companies themselves. There are many Church leaders and Moslem clerics who command large crowds for which entrepreneurs can approach with their brilliant ideas to sell to its members for the pooling of resources overtime. To achieve these several things are needed. One is credible entrepreneurs in the eyes of the public or social networks, the regulations that governs this mode of funding to protect the public are needed and the commitment on the part of the entrepreneur or managers of the fund to ensure the operationalization of the enterprise entrusted in their care. Finally, the patience on the part of contributors to allow the funds to grow in a safe asset and the willingness of entrepreneurs to trade sole ownership for joint ownership.

Obtaining real life data to fit exactly the description of this mode of enterprise was the main challenge with the study. Consequently, pseudo-simulated data which combines real life data was deployed in addition to carefully chosen hypothetical data that reflect the operations of private Senior High in Ghana. This opens a direction for further research, namely the quest for an enterprise whose formation fits model descriptions and how the models will perform under these circumstances. The study did not explore the effects of higher interest rate on the profitability beyond the band [0.05, 0.07] that were used for the analysis. In addition, reduction in fees beyond $3600 per year has also not been explored.

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