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--Dr. Mark Kimathi------------
--Machakos University---------
--P.O. Box 136 -90100---------

Dear Sir/Madam,

RE: INVITATION TO AN INTERNATIONAL CONFERENCE ON SCIENCE, TECHNOLOGY AND INNOVATION FOR SUSTAINABLE DEVELOPMENT IN DRYLAND ENVIRONMENTS

Umma University in Kajiado County, South Eastern Kenya University (SEKU) in Kitui County, Lukenya University in Makueni County and Machakos University in Machakos County, together with other partners, are jointly organizing an international Conference entitled *"Science, Technology and Innovation for Sustainable Development in Dryland Environments"* to be held on 19th-23rd November 2018. The theme of the conference is *"Harnessing Dryland Natural Resources for Sustainable Livelihoods in the Era of Climate Change*". The conference will be two-phased with a two day pre-conference training workshop on 19th-20th November 2018 at SEKU and the main conference on 21st-23rd November 2018 at Umma University. The conference will provide an excellent platform for the academia from around the world to engage with the industry, innovators, policy makers, value chain developers, farmers, and service providers among others so that higher education in Africa contributes to solving the problems of natural resources governance in the era of climate change.

We are therefore pleased to invite you to attend the pre-conference training workshop at SEKU Main Campus in Kitui on 19th-20th November 2018 and the Main conference at Umma University on 21st to 23rd November 2018. Please note that you will be responsible for your travel and accommodation arrangements and conference registration fee.

Yours Sincerely,

DR. ALI ADAN ALI
FOR THE: VICE-CHANCELLOR

Effects of Magnetic Induction on MHD Boundary Layer Flow of Dusty Fluid over a Stretching Sheet

November 12, 2018

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Abstract

This research investigates the effects of induced magnetic field on the MHDboundary layer laminar flow of a dusty fluidover a stretching flat plate surface. The fluid flow is influenced by an external uniform magnetic field applied perpendicular to the stretching plate. Moreover, the problem studiedis characterized by the parameters: Hartmann number, interactions with dust particles, stock resistance, Reynolds number and magnetic Reynolds number. The basic governing partial differential equations are transformed into a system of ordinary differential equations by applying suitable similarity transformations. These equations are in turn solved numerically by using collocation methodthrough MATLAB functions bvp4c. The effects of the characterizing parameters in this work on skin friction, dusty particles and magnetic induction are respectively tabulated and graphically depicted, then discussed. These results indicate the possible technological applications of the study in liquid based systems involving stretchable materials such as paintspray in motor vehicles. Although some of the numerical solutions agree with a number of the available results from previous studies, other results differ because of the effect of the induced magnetic field introduced in this study.

Key Words: Induced Magnetic field, Differential Equations, Dusty Fluid Flow, Skin Friction

List of Symbols

R_e Reynolds number.
B_o Induced magnetic field.
η_o Magnetic divisibility.
h' Magnetic induction.
g Dust particle velocity.
U_p Dust phase velocity.
u_w Stretching velocity.
C_f Skin friction coefficient.
η Similarity variable.
ν Kinematic viscosity.
U_∞ Free stream fluid velocity.
γ stock resistance.
f Dimensionless stream function for fluid velocity

1 Introduction

The problem of boundary layer dusty fluid flow has been under investigation over many years ago. Both unsteady and steady incompressible fluid flow has been studied by different researches such as Sakiadies (1961), Girresha (2012) and Mudassar (2017) amongst others. Recently, the study of steady dusty fluid flow under electrically conducted fluid flow has become a subject to the field of applied mathematics and above all with an introduction of magnetic field. This has enabled numerous researchers to study the effect of physical characterizing parameters such as Magnetic parameter, fluid particles parameters and dust particles parameters amongst others on dust fluid flow velocity profile, heat transfer, temperature and pressure profile. Nevertheless, these researchers limited themselves on the influence of applied external magnetic field to fluid flow at different angles without dwelling in any way with the effect of induced magnetic field on the fluid flow nor on the skin friction Moreover, despite the efforts of those researcher, no one attempted to carry out an investigation on the effects of physical characterizing parameters on induced magnetic profile such as magnetic Reynolds number (Rm). Therefore, to bridge this gap, the current study comes out to investigate the effect of induced magnetic field on MHD boundary layer dusty fluid flow over a stretching flat plate surface by introducing magnetic induction equations as a part of the equations governing dusty fluid flow. Henceforth, through that, we shall determine the effects of fluid flow parameters such as magnetic Reynolds number (Rm) and Reynolds number (Re) on induced magnetic field.

As different researchers have investigated the problem of boundary layer fluid flow and heat transfer over a stretching surface, this paper presents a few of their findings in order to establish the gap between them and the current study. By evaluating their investigations, this study aims at bringing more light to their investigations by introducing magnetic induction equation on the governing equations of the boundary layer steady incompressible dusty fluid flow, a key field of study which has drawn numerous interests to current mathematicians.

Mainly, the study of an electrically conducted boundary layer fluid flow over a stretching surface has been a subject of special interest for the first couple of decade due to its two phase nature namely, the phenomena occurring in a fluid both liquid or gas containing a distribution of solid spherical particles. Initially, boundary layer fluid flow was began by Prandtl (1904) and Sakiadics (1961) who studied boundary layer flow over a stretching surface moving with a constant velocity. He formulated a boundary layer equation for two-dimensional and axisymatic flow by using both exact and numerical methods through which he obtain the results. Saftman (1962) extended Sakiadies study by initiating a study of laminar dusty fluid flow which was improved by Crane (1970) by investigating two dimensional steady flow of an incompressible viscous fluid caused by a linearly stretching plate. More so, Vajraveru and Nayfeh (1992) discussed in depth the analysis of hydromagnetic flow of a dusty fluid flow over a stretching surface in the presence of suction velocity. On the same ground, Chakrabarti and Gupta (1992) discussed the hydromagnetic flow and heat transfer of a non-Newtonian fluid over a stretching surface. Thus many more authors studied the problem of MHD and viscous dissipation effects on heat transfer analysis. For instance, Chen (2008), basing himself on the above studies of boundary layer dusty fluid flow; he analyzed the mixed convection of a power law fluid over a stretching surface in the presence of thermal radiation and magnetic field. However, these studies despite their efforts of investigating, none of them investigated the relationship between dusty fluid flow and induced magnetic field, a fact that co-exists in any electrically conducted boundary layer dusty fluid flow passed a transverse magnetic field.

Due to the above noted loophole, more mathematicians continued with investigations on boundary layer fluid flow and heat transfer on a viscous and incompressible fluid induced by a stretching surface. G. J Gireesha et.al (2011) carried a study on the effect of hydrodynamic laminar boundary layer flow and heat transfer of a dusty fluid over an unsteady stretching surface in the presence of non-uniform heat generation and absorption. He studied the effect of various governing parameters such as magnetic parameter, heat source parameter, number density, fluid particles interaction parameter and unsteadiness parameter amongst others. Through his study, he noted there was an effect of magnetic parameters (M) on velocity distributions for fluid and dusty phase while other parameters remained fixed. He observed that, the momentum boundary layer thickness decreased as magnetic parameters increased and hence induces an increase in the absolute value of the velocity gradient at the surface. This reality revealed that the velocity profile decreased with an increase in the values of unsteadiness parameter and temperature component for both fluid and dusty phase decreased with an increase of fluid particles interactions parameters. Likewise, the study revealed that according to heat transfer, thermal boundary layers thickness was decreasing with an increase of Prandtl number.

In addition, B.J Gireesha et.al (2012) also studied boundary layer steady fluid flow by carrying out a study on magnetohydrodynamics boundary layer flow and heat transfer characteristics of a dusty fluid flow over a flat stretching sheet in viscous dissipation. Numerically, by applying Runge Kutta Fehlberg forth-fifth order method (RKF 45 method), he noted that the increasing Chandrasekhar number (Q) clearly escalated the magnitude of Lorentz hydromagnetic body force which retarded the flow whilst in the non-dimensional particles velocity was constant. This confirmed that the transverse magnetic field contributes to the thickening of thermal boundary layer. For instance, an applied transverse magnetic field produces a body force namely Lorentz force which oppose the motion by enhancing temperature. Thus, he concluded external magnetic field should be mild for effective cooling of stretching surface. Moreover, Runu S. et.al (2015) carried out a study on laminar boundary layer flow and heat transfer of a dusty fluid flow over a vertical permeable stretching surface. Through this investigation, he studied effect of physical parameter such as fluid particles interaction parameter, suction parameter, Prandtl number and Eckert number on the flow and heat transfer characteristics. This enabled him to discover that there was no significance change in the fluid phase velocity (U) unlike in the particles phase velocity (U_p) which increased with the increase of fluid particles interaction parameters (β). As well for the higher Prandtl number fluid had a thinner thermal boundary layer which increased the gradient of temperature and the surface heat transfer. Consequently he noted that fluid velocity (U) decreased asymptotically thought with a significant increase in dust particle phase velocity (U_p) due to increase of suction parameter (f_o). Recently, Mudassar J. et.al (2017) in view of the above previous studies carried out a study on an exact solution of MHD boundary flow of dusty fluid flow over stretching surface in the presence of applied magnetic field. By specifying his problem in terms of characterizing parameters known as fluid particles interaction parameters, (β) Magnetic field parameter (M) and mass concentration of dust particles parameter, (γ) he observed that both fluid and dust particles velocities decreased with an increase of magnetic parameter which increased Lorentz force resulting to the decrease of fluid velocity. As well, he noted the increase of physical parameters affected skin friction by increasing it which in turn produced a resistance to fluid flow.

Significantly, due to wide applications of MHD boundary layer flow in mechanical engineering, this study will play a great role in the field of mathematical application in the modern world. For instance, it will help in the field of cooling of reactors, electrostatic precipitations, power generations, MHD pumps, petroleum industries and designing of heat exchanger amongst others due to the application of magnetic induction equation. Moreover, due to emergence of great adventure in physics, this paper will also play an important role in the applications of geophysics, astrophysics, solar physics, meteorology and in the motion of earths' core (Mudassar 2017). Through the findings of this study, also those who work in the field of physics; they will find it easy to understand the whole concepts of steady dusty flow in relations to MHD boundary flow. Distinctively, this study will help many researchers in the field of MHD boundary layer flow to understand more the concepts of fluid flow embedded with dust particles as it is encountered in different engineering problems concerned with nuclear reactors cooling, powder technology and in paint spraying due to the application of induced magnetic field. Henceforth, this study will also enhance our understanding of dusty particles applications in the boundary layer especially as it is found in a cloud during nuclear explosions and in the technical processes.

As noted early, within the course of time the study of two-dimensional boundary layer dusty fluid flow where solid spherical particles are distributed in a fluid has become of great interest in a wide range of technical problems such as fluid flow through packed beds, sedimentation and centrifugal separations of particles. Mainly, this has been enhanced by two phase nature of dusty fluid flow whose occurrence contains the distribution of particles. Consequently, animated by the interest of two-dimensional boundary layer dusty fluid flow; researchers also worked out the dusty model for various flow configuration under the boundary layer fluid flow conditions by using different physical characterizing parameters. However, in spite of introducing external magnetic field in the MHD dusty fluid flow, previous researchers did not address effect of induced magnetic field in relation to dusty fluid flow over the stretching surface and skin friction. For instance, they limited themselves on the effects of physical characterizing parameters on skin friction and on fluid flow velocities. Therefore, by considering the effect of magnetic Reynolds number on induced magnetic profile as an additional parameter to previous studies, this paper extend previous investigation on dusty fluid flow by taking into account the effect of induced magnetic field on fluid flow, a reality that was previously overlooked. To achieve that, this study introduces magnetic induction equation on the two-dimensional steady incompressible MHD boundary layer dusty fluid flow over a stretching surface in order to study the effect of induced magnetic field on dusty fluid flow and on skin friction.

2 Equations governing the flow

2.1 Description of the flow problem

By Considering a steady flow of two dimensional laminar boundary layer flow of an electrically conducted viscous incompressible dusty fluid over a semi-infinite surface, let the surface be stretching with a velocity $U_w = cx$, where the positive constant c is the stretching rate. Let also the Cartesian coordinate system be located in X - axis and Y - axis along and normal to the surface respectively. Henceforth, let the origin of the system be at the leading edge as shown below in figure 2.1

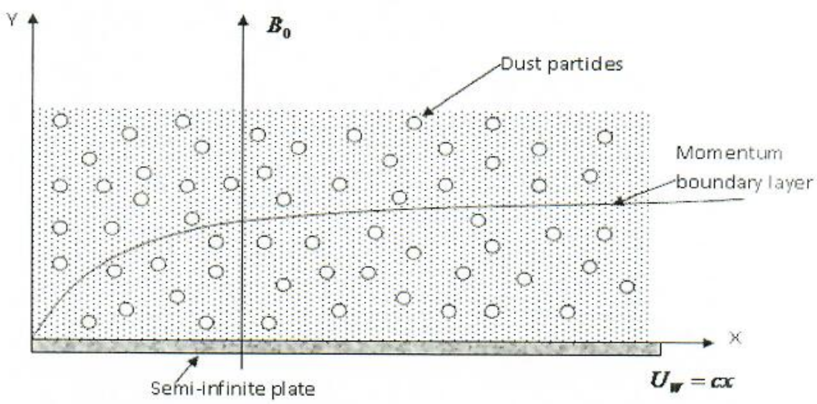

Figure 1: Geometry of the considered fluid flow indicating the dust particles, boundary layer and the externall applied magnetic field.

Deriving the governing equations, the Stokesian drag force is considered for the interaction between fluid and dust particles phase. More so, induced magnetic field is considered unlike external electric field which is negligible as a result of polarization of charges within the fluid flow. Additionally, for both fluid and dust particles, clouds are considered to be static at the beginning and pressure gradient is neglected meaning it is assumed to be constant. The dust particles are also assumed to be uniform in size and spherical in shape as appears in figure 2.1 above. Finally, number density of the dust particles is taken as constant throughout the flow whilst within the dust particles equation, both viscous force and magnetic field effects are neglected. Moreover, since the fluid being studied is electrically conducted and moving with the plate, then there is an induced magnetic field. The induction of magnetic field occurs leading to the formation of Alfve'n waves, whereby the motion of fluid drags the induced magnetic field (B_o) along the x - axis towards the direction of fluid flow. This dragging creates a force known as Lorentz force that counteracts the dragging of magnetic field. This happens repeatedly thus creating the induced magnetic field.

2.2 General governing equations

The following assumptions are considered in derivation of the specific fluid flow equations:

1. The flow is steady and incompressible.
2. Pressure gradient is negligible.
3. The number density is constant.
4. External electrical field (E) is negligible.
5. The dust particles are assumed to be spherical in shape and uniform in size.

On the other hand, the following assumptions were considered in derivation of the dust particles equations:

1. The flow is steady and incompressible.

2. There is no viscous force.

3. The magnetic field effect is negligible.

4. The pressure gradient is constant and negligible.

Therefore the general governing equations of the dusty fluid flow with induced magnetic field are:

$$(\vec{\nabla} \cdot \vec{U}) = 0 \tag{1}$$

$$(\vec{U} \cdot \vec{\nabla})\vec{U} = -\frac{1}{\rho}\vec{\nabla}P + \frac{\mu}{\rho}(\nabla^2 \vec{U}) + \frac{1}{\rho}(\vec{J} \times \vec{B}) + \frac{\nu}{\tau}(\vec{U_p} - \vec{U}) \tag{2}$$

$$(\vec{U}_p \cdot \vec{\nabla}) = 0 \tag{3}$$

$$(\vec{U}_p \cdot \vec{\nabla})\vec{U}_p = -\frac{1}{\rho}\vec{\nabla}P + \frac{\mu}{\rho}\nabla^2 \vec{U}_p + \frac{1}{\rho}(\vec{J} \times \vec{B}) + \frac{\nu}{\tau}(\vec{U} - \vec{U}_p) \tag{4}$$

$$\frac{\partial \vec{B}}{\partial t} = \vec{\nabla} \times (\vec{U} \times \vec{B}) + \eta_o \vec{\nabla}^2 \vec{B} \tag{5}$$

By considering the above stated assumptions together with the fluid flow description given in section 2.1 we have:

$$\vec{U} = u\hat{i} + v\hat{j} + 0\hat{k}, \quad \vec{B} = b\hat{i} + B_o\hat{j} + 0\hat{k}, \quad \vec{U_p} = u_p\hat{i} + v_p\hat{j} + 0\hat{k} \tag{6}$$

Since we know that pressure gradient is negligible from our assumptions, then:

$$-\frac{1}{\rho}\vec{\nabla}P = 0 \tag{7}$$

2.3 Specific Governing equations

Using (7) and the component terms in (6) above, in the given general equations, we obtain the specific equations below.

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \tag{8}$$

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = \frac{\mu}{\rho}(\frac{\partial^2 u}{\partial y^2}) + \frac{\delta}{\rho}(B_o vb - uB_o^2) + \frac{\nu}{\tau}(u_p - u) \tag{9}$$

$$\frac{\partial u_p}{\partial x} + \frac{\partial v_p}{\partial y} = 0 \tag{10}$$

$$(\vec{U}_p\frac{\partial u_p}{\partial x} + \vec{V}_p\frac{\partial u_p}{\partial y}) = \frac{1}{\tau}(\vec{u} - \vec{u}_p) \tag{11}$$

$$\frac{\partial}{\partial y}(vb - uB_o) + \eta_o\frac{\partial^2 b}{\partial y^2} = 0 \tag{12}$$

Since $b = b(y)$ and B_o is a constant.
The boundary conditions accompanying the above specific governing equations are:

$$y = 0; \quad u = u_w(x), \quad v = 0, \quad b = B_o;$$

$$y \to \infty; \quad u = 0, \quad u_p = 0, \quad v_p = v; \quad b = 0$$

3 Method of Solutions

3.1 Similarity Transformations of the Specific Equations and Boundary conditions

The following similarity relations are used to transform the specific equations in section 2.3 and boundary conditions:

$$\eta = \sqrt{\frac{c}{\nu}}y; \quad U_p = cxg'(\eta); \quad V_p = -\sqrt{c\nu}g(\eta)$$

$$b = cxh'(\eta); \quad u = cxf'(\eta); \quad v = -\sqrt{cx}f(\eta)$$

Transformation of the boundary conditions in section 2.3 for $y = 0$ and $y \to \infty$ respectively yields,

$$\eta = 0; \quad f' = 1; \quad f(0) = 0$$

$$\eta \to \infty; \quad g = f; \quad f'(\infty) = 0; \quad h'(\infty) = 0; \quad g'(\infty) = 0$$

The transformation of specific governing equations in section 2.3 yields the following ordinary differential equations:

$$f''' + ff'' - f'^2 - \beta\gamma(g' - f') - M(-\frac{\sqrt{c\nu}}{B_o}fh' + f') = 0 \tag{13}$$

$$gg'' - g'^2 + \beta(f' - g') = 0 \tag{14}$$

$$h''' - \frac{R_m}{R_e}fh'' - \frac{R_m}{R_e}f'h' - \frac{B_o}{\sqrt{c\nu}}\frac{R_m}{R_e}f'' = 0 \tag{15}$$

As a result of the transformations, the non-dimensional parameters defined below are obtained:

Fluid particles interaction parameter,$\beta = \frac{1}{c\tau}$, Magnetic parameter,$M = \frac{\delta B_o^2}{c\rho}$. Skin friction coefficient, $C_f = \frac{-\tau_w}{\rho u_{w^2}}$, and Magnetic Reynolds number,$Rm = \frac{V_o L_o}{\eta}$.

3.2 Numerical Methods of solution

In order to perform the numerical approximations the ordinary differential equations obtained in section 3.1 are first reduced from their respective higher order to first order. Letting $z_1 = f; \quad z_2 = f'; \quad z_3 = f''$, results to the following system of first order differential equations, for the equation (13):

$$\begin{aligned} z_1' &= z_2 \\ z_2' &= z_3 \\ z_3' &= -z_1 z_3 + z_2^2 - \beta\nu z_5 - z_2 + M z_2 \end{aligned} \tag{16}$$

Further, letting $z_4 = g; \quad z_5 = g'$ yields the following system of differential equations, for equation (14):

$$\begin{aligned} z_4' &= z_5 \\ z_5' &= \frac{z_5^2}{z_4} - \frac{\beta}{z_4}(z_2 - z_5) \end{aligned} \tag{17}$$

Finally, letting $z_6' = h; \quad z_7 = h', \quad z_8 = h''$ gives rise to the system below, for equation (15):

$$\begin{aligned} z_6' &= z_7 \\ z_7' &= z_8 \\ z_8' &= \frac{R_m}{R_e} \cdot z_1 z_7 + \frac{R_m}{R_e} z_2 z_7 + \frac{B_o}{\sqrt{c\nu}} \cdot \frac{R_m}{R_e} z_3 \end{aligned} \tag{18}$$

Therefore the equations (13), (14), and (15) now becomes a system of equations of the form:

$$z' = F(\eta, z)$$

where:

$$z = \begin{bmatrix} z_1 \\ z_2 \\ z_3 \\ z_4 \\ z_5 \\ z_6 \\ z_7 \\ z_8 \end{bmatrix} \quad F = \begin{bmatrix} z_2 \\ z_3 \\ -z_1 z_2 + z_2^2 - \beta\nu z_5 - z_2 + M z_2 \\ z_4 \\ z_5 \\ \frac{z_5}{z_4} - \frac{\beta}{z_4}(z_2 - z_5) \\ z_6 \\ z_7 \\ z_8 \\ \frac{R_m}{R_e} \cdot z_1 z_7 + \frac{R_m}{R_e} z_2 z_7 + \frac{B_o}{\sqrt{c\nu}} \cdot \frac{R_m}{R_e} z_3 \end{bmatrix} \tag{19}$$

4 Discussions of the Simulation Results

4.1 Effects of induced magnetic induction on skin friction

A collocation method was used to solve the above first order systems of ordinary differential equations through the MATLAB function *bvp4c*. As a result, we obtained the relationship between skin friction and different dimensionless parameters used in this study as shown in Table 1.

Table 1: Skin friction in relation to different dimensionless parameters

No.	M	β	γ	Rm	Re	C_f
1	0.3	0.5	2×10^{-3}	30	3500	-1.0228
2	0.6	0.5	2×10^{-3}	30	3500	-1.1231
3	1.0	0.5	2×10^{-3}	30	3500	-1.2338
4	0.6	0.1	2×10^{-3}	30	3500	-1.1232
5	0.6	0.7	2×10^{-3}	30	3500	-1.1230
6	0.6	0.5	1×10^{-3}	30	3500	-1.1208
7	0.6	0.5	3×10^{-3}	30	3500	-1.1248
8	0.6	0.5	2×10^{-3}	10	3500	-1.1228
9	0.6	0.5	2×10^{-3}	50	3500	-1.1234
10	0.6	0.5	2×10^{-3}	30	1500	-1.1244
11	0.6	0.5	2×10^{-3}	30	5500	-1.1231

From Table 1 above, the effects of Reynolds number (Re) and magnetic Reynolds number (Rm) on skin fiction are presented for the first time unlike other parameters found in the previous studies such as Hartmann number (M), fluid particles interaction parameters (β) and stock resistance (γ). In particular, presents effect of Reynolds number and Magnetic Reynolds number on skin friction as found in equation (15). The effects of Reynolds number (Re) and fluid particles interaction parameters (β) on skin friction shows a direct proportionality, meaning their increase leads to an increase of skin frictions. For instance, an increase of Reynolds Number leads to an increase in inertia forces, that in turn causes an increases in skin friction between fluid flow and the stretching surface. Likewise, from Table 1, there is an inverse proportionality between Hartmann number (M), magnetic Reynolds number and skin friction coefficient. The increase of Hartman number (M) and magnetic Reynolds number decreases skin friction. This means, the increase of these two parameters retards the fluid flow and hence reduces the skin friction which in turn reduce stickiness of dust particles to the stretching surface. This observations confirms that, since all the three equations are coupled, there is an effect of induced magnetic field on skin friction and fluid flow.

4.2 Effect of physical characteristic parameters on fluid flow profile and on induced magnetic profile

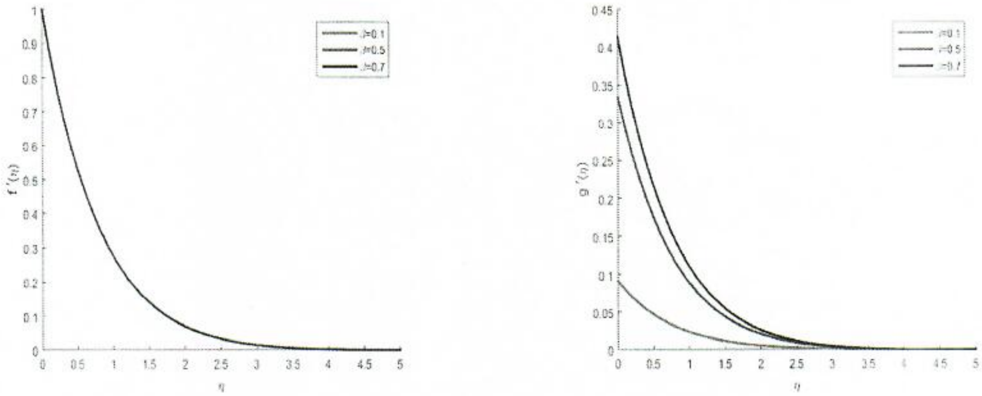

Figure 2: *Left*: Fluid particles velocity profile for various values of β. *Right*: Dust particles velocity profile for various values of β

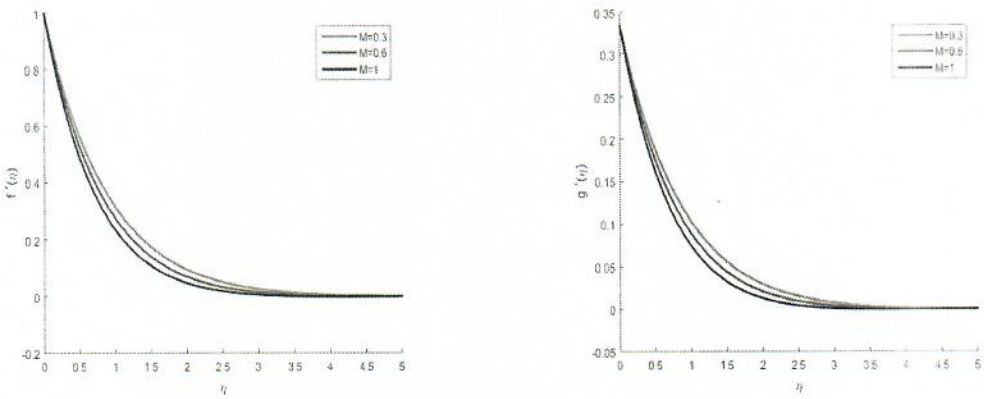

Figure 3: *Left*: Fluid particles velocity profile for various values of M. *Right*: Dust particles velocity profile for various values of M

The variation of parameters such as Hartmann number (M), fluid particles interaction parameters (β) and stock resistance (γ), Reynolds number (Re) and Magnetic Reynolds number (Rm) produced results which partially agreed with previous investigations. In the simulations, the Reynolds number (Re) , Magnetic Reynolds number(Rm) and dust particles interaction parameter (β) had no significant effects on fluid velocity profile. Additionally, Re, Rmhad no effect on dust particles velocity. However, figure 2 and 3 indicates clearly that fluid particles interaction parameter (β) and Hartmann number (M) have significant effects on dust particle and fluid velocity profiles. For instance, the increase of Hartmann number (M) leads to retardation of flow and hence a decrease of both fluid and dust particles velocities. An increase of β clearly signifies an increase in the dust particles velocity due to increased interactions of the particles. It should be noted that the increase of Hartmann (M) leads to decrease of dust particles velocity due to the fact that an increase of Hartmann number produces an opposing Lorentz forces on dust particles velocity as observed in Mudassar (2017).The variations of fluid particles interaction parameter (β) and Hartmann number (M) do not have significant effect on the magnetic induction profile.

In figure 4 we observe that by increasing stock resistance (γ) and Reynolds number (Re) parameters, the values of magnetic induction profile decreases. However, figure 4 depicts a direct proportionality between magnetic Reynolds number (Rm) and magnetic induction profile. Increase of magnetic

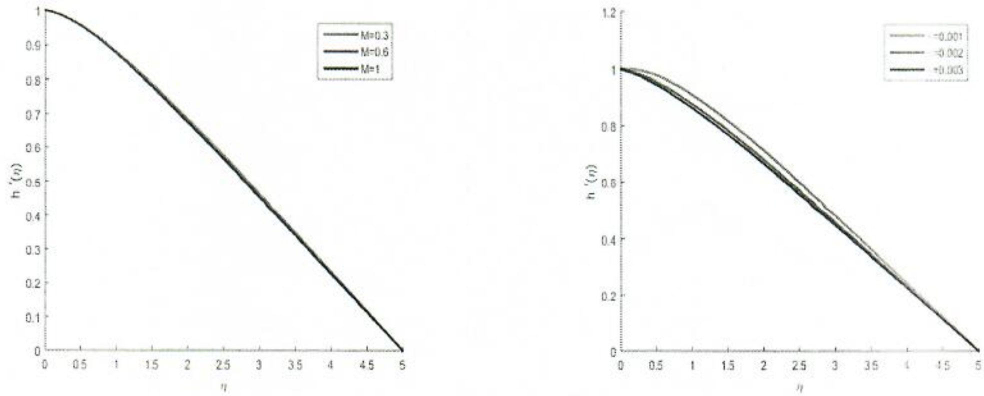

Figure 4: *Left*: Induced magnetic profile for various values of M. *Right*: Induced magnetic profile for various values of γ

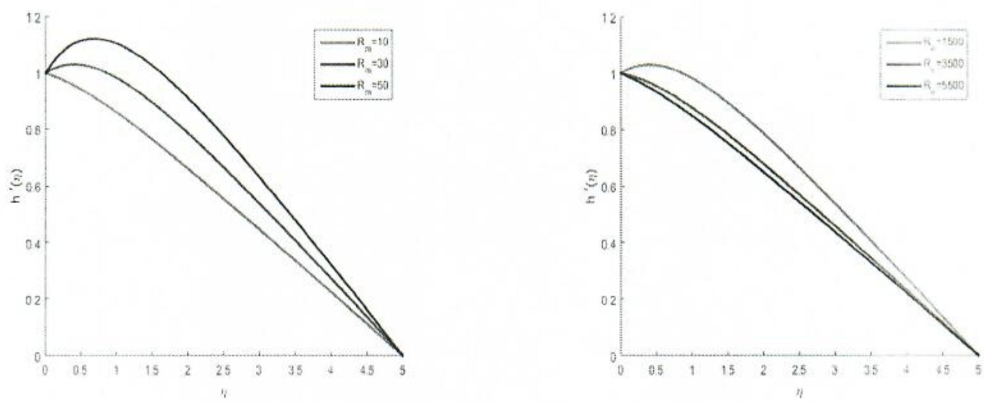

Figure 5: *Left*: Induced magnetic profile for various values of Rm. *Right*: Induced magnetic profile for various values of Re

Reynolds number (Rm) leads to an increase of magnetic induction profile due to the fact that the presence of induced magnetic field (B_o) normal to the fluid flow in an electrically conducting fluid produces a Lorentz force which acts against the fluid flow. An increase in Re significantly increases the fluid velocity such that the Lorentz force generated to counteract the motion is weakened by the sustained high fluid velocity.

5 Conclusion

The effect of various fluid flow physical parameters on fluid and dusty particles velocities, on induced magnetic field and on skin friction coefficient has been studied numerically. These parameters included Hartmann number (M), fluid particles interaction parameters (β), stock resistance (γ), Reynolds number (Re) and magnetic Reynolds number (Rm). The results of different values of these parameters in relations to skin friction coefficient, velocity profile, dust particles profile and magnetic induction profile were depicted through a table and graphs respectively. These results presented some agreement with those obtained by Gireesha (2012) and Mudassar (2017) in their investigations especially as per the common parameters that we used. For instance just like in the study, then above scholars observed that, dust particles velocity increased with the increase of fluid particles interactions (β) which produced a resistance force to the fluid flow hence an increase of skin friction. Henceforth, in the light of this study, the following conclusions are drawn.

1. The increase of Hartmann number and magnetic Reynolds number leads to reduction of skin

friction. This shows there is an effect of induced magnetic field on skin friction and shear stress, a reality that leads to total reduction of stickiness of dust particles on the stretching surface. Also this indicates an effect of magnetic induction field on dusty particles fluid flow.

2. The increase of magnetic parameter (M) leads to decrease in both fluid and dust particles velocities mainly because the increase of magnetic field induces more opposing Lorentz forces which results to reduction of both fluid and dust particles velocities.

3. The magnetic Reynolds number (Rm) and Reynolds number (Re) have no significant effects on both velocity profile and dust particles profile unlike in the dusty particles phase profile (U_p) which increased with the increase of fluid particles interaction parameter (β). This effect contributes to decrease of momentum boundary layer thickness because as magnetic parameter (M) increase, it induces an increase in absolute value leading into reduction of fluid flow velocity.

Therefore, through these conclusions, deduces that there is a substantial effect of induced magnetic field on MHD boundary layer dusty fluid flow. Hence, it is our conclusion that the application of magnetic induction is significantly important in engineering processes such as in the cooling of reactors, petroleum industry and paint spray such as in motor vehicles whereby a reduction of skin friction within the stretching surface is required in order to make stretching surfaces shinny for the sake of overcoming stickiness of dust.

5.1 Recommendations for further research

Based on the finding of this study, the researcher observed that there were still some areas of mutual concern. Henceforth, animated by this study, the researcher made the following recommendations for future investigations.

1. Effects of an inclined magnetic induction on MHD dusty boundary Layer flow over an inclined stretching sheet.

2. Analytical solutions of an MHD boundary layer flow of a dusty fluid over a parallel flat permeable stretching surface.

References

[1] B.J Gireesha, B. J., Roppa, G. S., Lokesh, H. J., Bagewadi, C. S. (2012). MHD flow and heat transfer of a dusty fluid over a stretching sheet, International Journal of Physical and Mathematical Sciences, Vol. 3, Pp. 171-182.

[2] Gireesha, B. J., Ramesh, G. K., Lokesh, H. J., and Bagewadi, C. S. (2011). Boundary layer flow and heat transfer of a dusty fluid over a stretching vertical surface, Appl. Mathe., Vol. 2, Pp 475-481.

[3] Roopa, B. J., Gireesha, G. S., and Bagewadi, C.S. (2012). Effect of viscous dissipation and heart source on flow and heart transfer of dusty fluid over unsteady stretching sheet. Appl. Math. Pp 1001-1014.

[4] Gireesha, B. J., Ramesh, G., Subbas-Abel, M., and Bagewadi, C.S (2011). Boundary layer flow and heat transfer of a dusty fluid flow over a stretching with non-uniform heat source/sink, International Journal of multiphase flow, 37(8), 977-982.

[5] Ramesha, G. K., Gireesha, B. J., and Bagewadi,C. S. (2012). Heat transfer in MHD dusty boundary layer flow over an inclined stretching sheet with non-uniform heat source/sink, Advances in mathematics physics, India: doi:1155/2012/65-7805.

[6] Attia, H. A. (2002). Unsteady MHD flow and heart transfer of dusty fluid between parallel plates with variable physical properties, Appl. Math. Model, 26, Pp. 864-875.

[7] Anderson, H. I., Bech, K. H., and Dandapart, B. S. (1992). Magnetohydrodynamics flow of a power-law flow over a stretching sheet, International Journal of Non-linear mechanics, Vol. 27, no. 6, Pp. 929-936.

[8] Crame, J. (1970). Flow past a strecting plate, Zeitschrisft ful angewandte Mathematic undphyisik ZaAMP, VOL. 21, NO. 4, Pp. 645-647.

[9] Jalil, M., Asghar, S., and Yasmeen, S. (2017). An exact solution of MHD boundary layer flow of dusty fluid over a stretching surface, *Published online April 2017, https:doi.org/10.1155/2017/2307469.*

[10] Jalil, M., and Asghar, S. (2013). Flow of Power- law fluid over a stretching surface: a lie group analysis, International Journal of non- linear Math. Vol.39 No.8, Pp.65-71.

[11] Raisinghan, M. D. (1939). Fluid dynamics with complete hydrodynamics and boundary layer theory, S. Chand and Company, Ramnagar.

[12] M. K Jain, M. K., Lyengar, S. R. K., and Jain, R. K. (1984). Numerical methods for scientific and engineering computation (Sixth Edition), New age International Limited, New Delhi.

[13] Data, N., and Mishra, S. K. (1982). Boundary layer flow of a dusty fluid over a semi- infinite flat plate, Acta. Mechanical. Math., Vol.1 No.1-2, Pp.71-83..

[14] Vajravelu, K., and Nayfeh, J. (1992). Hydromagnetic flow of a dusty fluid over a stretching sheet, International Journal of non-liner Mechanics, Vol. 27. No. 6, Pp 937-945.

[15] Snigdha, S., and Sharma, G. S. (1987). Unsteady flow of an electrically conducting dusty viscous liquid between two parallel plates, India J. pure Appl. Math., 189(12) Pp. 1131-1138.

[16] Sharidah, S., Mahmood, T., and Poo, T. (2008). Similarity solutions for the unsteady boundary layer flow and heat transfer due to a stretching sheet, International Journal of Applied Mechanics and Engineering, Vol. 11. No. 3, Pp. 647-654.