MACHAKOS UNIVERSITY

University Examinations 2021/2022

SCHOOL OF PURE AND APPLIED SCIENCES

DEPARTMENT OF PHYSICAL SCIENCES

FIRST YEAR SUPPLEMENTARY/SPECIAL EXAMINATION FOR BACHELOR OF SCIENCE (TELECOMMUNICATION AND INFORMATION TECHNOLOGY

BACHELOR OF EDUCATION (SCIENCE)

BACHELOR OF SCIENCE IN MATHEMATICS

SPH 103: MATHEMATICS FOR PHYSICS

DATE: 15/3/2022 **TIME: 11:00-1:00PM**

INSTRUCTIONS:

- The paper consists of **two** sections.
- Section **A** is **compulsory** (30 marks).
- Answer any **two** questions from section **B** (each 20 marks).

QUESTION ONE (COMPULSORY)

a) The Cartesian coordinates of a point in the *xy* plane are; $(x, y) = (-3.50, -2.50)$. Find the polar coordinates of this point. (3 marks)

b) Calculate the volume V of a parallelepiped with sides $\vec{a} = \hat{i} + 2\hat{j} + 3\hat{k}, \vec{b} = 4\hat{i} + 5\hat{j} + 6\hat{k}$ and $\vec{c} = 7\hat{i} + 8\hat{j} + 10\hat{k}$. (4 marks)

c) Find the angle between the vectors; $\vec{a} = \hat{i} + 2\hat{j} + 3\hat{k}$ and $\vec{b} = 2\hat{i} + 3\hat{j} + 4\hat{k}$. (4 marks)

d) Find $\mathrm{f}^{-1}(x)$ when $\mathrm{f(x)} = 2\mathrm{x} + 1, \mathrm{x} \in \Re$ (3 marks)

e) Evaluate the limits;

(i) $\lim_{x \to 1} (x^2 + 2x^3)$, (1 mark)

(ii) $\lim_{x \to 0} (x \cos x)$, (1 mark)

(iii) $\lim_{x \to \pi/2} \left(\frac{\sin x}{x} \right)$ (1 mark)

f) Find the first derivative of; (i) $x^3 - 3xy + y^3 = 2,$ and (ii) $y = a^x$ with respect to x.

(4 marks)

g) Find the first and second partial derivatives of the function $f(x, y) = 2x^3 y^2 + y^3$.

(4 marks)

h) Show that if $\vec{A} = \vec{B} + \lambda \vec{C},$ for some scalar $\lambda,$ then $\vec{A} \times \vec{C} = \vec{B} \times \vec{C}.$ (2 marks)

i) Solve the equation $x^2 + 8x + 10$ by completing the square method. (3 marks)

QUESTION TWO (20 MARKS)

a) Find the points on the curve with equation $y = x^3 + 6x^2 + 5$ where the value of the gradient is -9. (4 marks)

b) The vertices of triangle **ABC** have position vectors ***a***, ***b*** and ***c*** relative to some origin 0 (see figure 2).

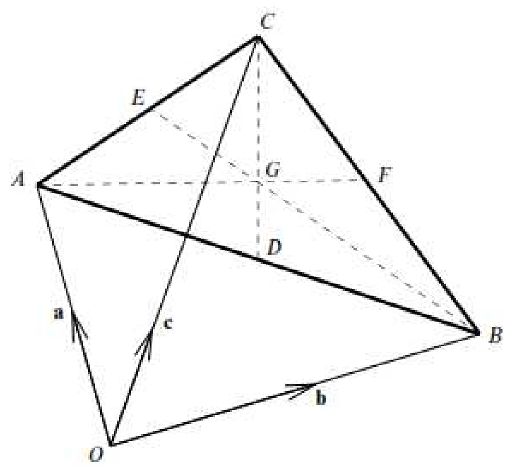

Figure 2.

Find the position vector of the centroid G of the triangle. (5 marks)

c) Using an Argand diagram, distinguish between the modulus and argument of a complex number. (3 marks)

d) Calculate the modulus and argument of the complex number $-1 + \sqrt{3i}$ and sketch the argand diagram. (4 marks)

e) Given that; $z_1 = 3 + 4i$ and $z_2 = 1 - 2i,$ find :

(i) $z_1 + z_2,$ (1 mark)

(ii) $z_1 - z_2,$ and (1 mark)

(iii) $z_1 z_2$ (1 mark)

QUESTION THREE (20 MARKS)

a) Express $\frac{1}{\left(-2+2\sqrt{3}i\right)}$ in the form $a+ib$. (6 marks)

b) Show that $\cosh 3x = 4\cosh^3 x - 3\cosh x$. (4 marks)

c) If $y = x^2 \tan^{-1} 2x$, find the $\frac{dy}{dx}$. (4 marks)

d) **(i)** Find $\int \left(2x^3 - 3x + 4\right)dx$ (2 marks)

(ii) Evaluate $\int_0^{\frac{3}{2}} \frac{dx}{\sqrt{9-x^2}}$. (4 marks)

QUESTION FOUR (20 MARKS)

a)

(i) Write down the polar form of a complex number**.** (1 marks)

(ii) Find $z_1 z_2$ if $z_1 = 2\left(\cos\frac{\pi}{3} + i\sin\frac{\pi}{3}\right)$ and $z_2 = 3\left(\cos\frac{\pi}{6} - i\sin\frac{\pi}{6}\right)$. (3 marks)

b) The functions f, g, and h are defined by:

$f(x) = 2x$ for $x \in \Re$; $g(x) = x^2$ for $x \in \Re$; $h(x) = \frac{1}{x}$ for $x \in \Re, x \neq 0$. Find the following.

(i) $fg(x)$ (ii) $gf(x)$ (iii) $gh(x)$ (iv) $f^2(x)$ (v) $fgh(x)$ (5 marks)

c) Find all the stationary points on the curve of $y = 2t^4 - t^2 + 1$ and sketch the curve.

(6 marks)

d) Express $\left(\sqrt{3}+i\right)^{10}$ in the form $a+ib$ (5 marks)

QUESTION FIVE (20 MARKS)

a) Express $2-2i$ in the form $re^{i\theta}$. (4 marks)

b) Express $\tan 4\theta$ in terms of $\tan\theta$. (6 marks)

c) A force of $\vec{F} = (2.0\hat{i} + 3.0\hat{j})N$ is applied to an object that is pivoted about a fixed axis aligned along the z-coordinate axis. The force is applied at a point located at $\vec{r} = \left(4.0\hat{i} + 5.0\hat{j}\right)m$. Find the torque $\vec{\tau}$ applied to the object. (5 marks)

d) The quartic equation $x^4 + 2x^3 + 14x + 15 = 0$ has one root equal to $1+2i$. Find the other three roots (5 marks)