MACHAKOS UNIVERSITY

University Examinations 2021/2022

SCHOOL OF PURE AND APPLIED SCIENCES

DEPARTMENT OF PHYSICAL SCIENCES

SECOND YEAR SUPPLEMENTARY/SPECIAL EXAMINATION FOR BACHELOR OF SCIENCE (TELECOMMUNICATION AND INFORMATION TECHNOLOGY)

SPH 205: MATHEMATICAL PHYSICS

DATE: 18/03/2022 **TIME: 11:00-1:00 PM**

INSTRUCTIONS:

- The paper consists of **two** sections.
- Section **A** is **compulsory** (30 marks).
- Answer any **two** questions from section **B** (each 20 marks).

QUESTION ONE (30 MARKS)

(a) A force of $\vec{F} = (2.0\hat{i} + 3.0\hat{j})N$ is applied to an object that is pivoted about a fixed axis aligned along the z-coordinate axis. The force is applied at a point located at $\vec{r} = \left(4.0\hat{i} + 5.0\hat{j}\right)m$. Find the torque $\vec{\tau}$ applied to the object. (3 marks)

(b) Find the area of a parallelogram with sides $\vec{a} = \hat{i} + 2\hat{j} + 3\hat{k}$ and $b = 4\hat{i} + 5\hat{j} + 6\hat{k}$. (2 marks)

(c) Find the volume *V* of the parallelepiped with sides $\vec{a} = \hat{i} + 2\hat{j} + 3\hat{k}, \vec{b} = 4\hat{i} + 5\hat{j} + 6\hat{k}$ and $\vec{c} = 7\hat{i} + 8\hat{j} + 10\hat{k}$. (2 marks)

(d) A particle moving in the *xy* plane undergoes a displacement given by $\Delta\vec{r} = \left(2.0\hat{i} + 3.0\hat{j}\right)m$ as a constant force $\vec{F} = \left(5.0\hat{i} + 2.0\hat{j}\right)N$ acts on the particle. Calculate the work done by $\vec{F}$ on the particle. (3 marks)

(e) **(i)** Find the gradient of the scalar field; $f(x, y, z) = xy^2 - yz$ (3 marks)

(ii) Find the divergence of the vector field $F = 3x^2\hat{i} - 6xy\hat{j}$. Explain the result (2 marks)

(iii) Find the curl of the vector field $F = y^3\hat{i} + xy\hat{j} - z\hat{k}$. (3 marks)

(f) The first three terms of an infinite geometric progress are; 16, 12 and 9.

(i) Write down the common ratio (1 mark)

(ii) Find the sum of the terms of the progression (2 marks)

(g) Evaluate the limits; $\lim_{x\to 1}\left(x^2 + 2x^3\right)$, $\lim_{x\to 0}\left(x\cos x\right)$, $\lim_{x\to \pi/2}\left(\frac{\sin x}{x}\right)$ (3 marks)

(h) Find the first and second partial derivatives of the function $f(x, y) = 2x^3y^2 + y^3$. (3 marks)

(i) Show that $I \equiv \int_{-1}^{1} U_n(x)U_n(x)(1-x^2)^{1/2}\,dx = \frac{\pi}{2}$. (3 marks)

QUESTION TWO (20 MARKS)

(a) Show that if $\vec{a} = \vec{b} + \lambda.\vec{c}$ for some scalar λ, then $\vec{a}\ \mathrm{x}\ \vec{c} = \vec{b}\ \mathrm{x}\ \vec{c}$. (3 marks)

(b) Find the direction of the line of intersection of the two planes;

$x + 3y - z = 5$ and $2x - 2y + 4z = 3$. (3 marks)

(c) Find the vector area of the surface of the hemisphere $x^2 + y^2 + z^2 = a^2, z \geq 0$, by evaluating the line integral $S = \frac{1}{2}\oint_C r\ \mathrm{x}\ dr$ around its perimeter. (6 marks)

(d) (i) Find the greatest number of terms required for the sum of; 4+9+14+19+…… to exceed 2000. (3 marks)

(ii) State the mathematical form of a power series. Determine the range of values of x for which the following power series converges: $P(x) = 1 + 2x + 4x^2 + 8x^3 + ...$ (5 marks)

QUESTION THREE (20 MARKS)

(a) A particle of mass m with position vector r relative to some origin O experiences a force F, which produces a torque (moment) $\tau = \vec{r}$ x $\vec{F}$ about O. The angular momentum of the particle about O is given by $\vec{L} = \vec{r}$ x m$\vec{V}$, where V is the particle's velocity. Show that the rate of change of angular momentum is equal to the applied torque. (3 marks)

(b) Show that $\nabla.(\nabla\phi \times \nabla\varphi) = 0$, where ϕ and φ are scalar fields. (4 marks)

(c) Find the vector area of the surface of the hemisphere $x^2 + y^2 + z^2 = a^2$ with $z \geq 0$. (6 marks)

(d) Plane polar coordinates, (ρ, ϕ) and Cartesian coordinates, (x, y) are related by the expressions; $x = \rho\cos\phi$, $y = \rho\sin\phi$. An arbitrary function $f(x, y)$ can be re-expressed as a function $g(\rho, \phi)$. Transform the expression $\frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2}$ into one in (ρ, ϕ). (7 marks)

QUESTION FOUR (20 MARKS)

(a) Find the element of the area on the surface of a sphere of radius a, and hence calculate the total surface area of the sphere. (7 marks)

(b) For a compressible fluid with time-varying position-dependent density $\rho(r, t)$ and velocity field $v(r, t)$, in which fluid is neither being created nor destroyed, show that; $\frac{\partial \rho}{\partial t} + \Delta.(\rho.v) = 0.$ (6 marks)

(c) Express the vector field $\vec{a} = yz\hat{i} - y\hat{j} + xz^2\hat{k}$ in cylindrical polar coordinates, and hence calculate its divergence. Show that the same result is obtained by evaluating the divergence in Cartesian coordinates. (7 marks)

QUESTION FIVE (20 MARKS)

(a) Show that the area of a region R enclosed by a simple closed curve C is given by $A = \frac{1}{2}\oint_C (xdy - ydx) = \oint_C xdy = -\oint_C ydx.$ Hence calculate the area of the ellipse $x = a\cos\phi,\ y = b\sin\phi.$ (5 marks)

(b) A ladder of length, *L* stands on level ground and can be leaned at any angle against a vertical wall. Find the equation of the curve bounding the vertical area below the ladder. (5 marks)

(c) Find the angle between the vectors; $\vec{a} = \hat{i} + 2\hat{j} + 3\hat{k}$ and $\vec{b} = 2\hat{i} + 3\hat{j} + 4\hat{k}$. (3 marks)

(d) Given the vector field $\vec{a} = y\hat{i} - x\hat{j} + z\hat{k},$ verify Stoke's theorem for the hemispherical surface $x^2 + y^2 + z^2 = a^2, z \geq 0.$ (4 marks)

(e) The Cartesian coordinates of a point in the *xy* plane are; $(x, y) = (-3.50, -2.50)$m. Find the polar coordinates of this point. (3 marks)