SCHOOL OF PURE AND APPLIED SCIENCES

DEPARTMENT OF PHYSICAL SCIENCES

FOURTH YEAR SUPPLEMENTARY/SPECIAL EXAMINATION FOR BACHELOR OF EDUCATION (SCIENCE) AND BACHELOR OF EDUCATION SCIENCE (SPECIAL NEEDS)

SPH 400: CLASSICAL MECHANICS

DATE: 15/03/2022 **TIME: 2:00-4:00 PM**

INSTRUCTIONS:

- The paper consists of **two** sections.
- Section **A** is **compulsory** (30 marks).
- Answer any **two** questions from section **B** (each 20 marks).

QUESTION ONE (30 MARKS)

a) State the three Newton's laws. (3 marks)

b) Differentiate between the following terms

i) Curvilinear and rectilinear motion (2 marks)

ii) Translational and rotational motion (2 marks)

iii) Center of mass and center of gravity (2 marks)

c) Define the following terms and give their mathematical expressions

i) Moment of inertia (2 marks)

ii) Linear momentum (2 marks)

iii) Potential energy (2 marks)

iv) Virtual work (2 marks)

d) Using the Langrage equation, show that the solution to the simple harmonic motion given in figure 1 is $x = A\cos\omega t$ (5 marks)

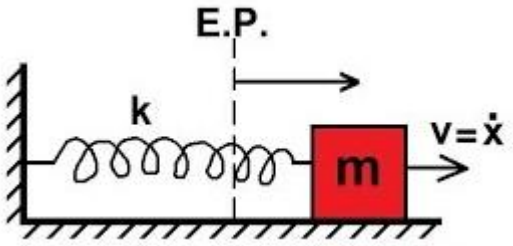

Figure 1

e) Plot the point whose polar coordinate is given by $\left(10, \frac{\pi}{6}\right)$ (5 marks)

f) Two particles P and Q of mass 1kg and 3 kg respectively start moving towards each other from rest under mutual attraction. Calculate the velocity of their center of mass. (3 marks)

QUESTION TWO (20 MARKS)

a) A projectile is launched with a speed of 100 m/s at an angle of 50° from the ground. Find the projectile's

(i) Total time in the air (2 marks)

(ii) Max height reached (2 marks)

(iii)Maximum horizontal distance covered (2 marks)

Given that it takes 6 seconds to hit the ground.

b) A trolley of mass 8kg is held at rest on a smooth inclined plane as shown in figure 2. When released, it moves down through a vertical height of 2.5m while accelerating. It then collides with a second trolley of mass 6kg which is at rest on a smooth horizontal plane. After collision, the two trolleys coalesce and move forward.

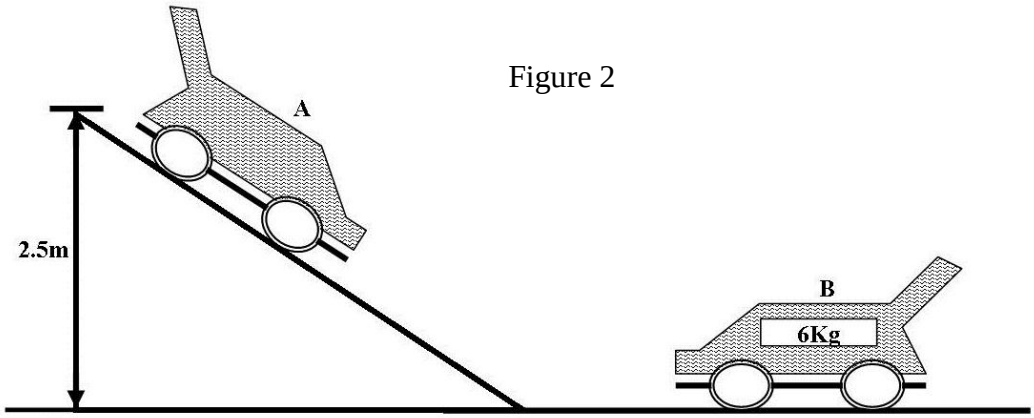

Figure 2

Calculate:

i) The velocity of *A* just before collision. (3 marks)

ii) The common velocity of the two trolleys after the collision. (3 marks)

iii) The kinetic energy just before and after the collision. Account for the difference in kinetic energy. (4 marks)

c) Figure 3 shows a force applied perpendicular to the handle of a spanner.

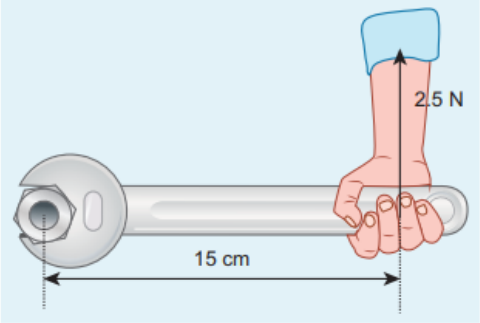

Figure 3

Find the:

(i) Torque exerted by the force about the center of the nut (2 marks)

(ii) Direction of torque (2 marks)

QUESTION THREE (20 MARKS)

a) Show that for a rigid body, virtual work is zero (4 marks)

b) State the:

i) Principle of conservation of angular momentum (2 marks)

ii) D'Alembert's principle of virtual work (2 marks)

iii) Hamilton's principle (2 marks)

c) A uniform disc of mass 100g has a diameter of 10 cm. Calculate the total energy of the disc when rolling along a horizontal table with a velocity of 20 cms^{-1}. (take the surface of table as reference) (6 marks)

d) A motor attached to a grindstone exerts a constant torque of 10 N-m. The moment of inertia of the grindstone is I = 2 $kg\text{-}m^2$. The system starts from rest. Calculate the kinetic energy after 8s (4 marks)

QUESTION FOUR (20 MARKS)

a) Figure 4 shows a ladder sliding down a wall. Find the equation of motion. (Assume the motion is in plane and the wall and the ladder are smooth. Where C(x,y) is the coordinates of the center of gravity of the ladder (10 marks)

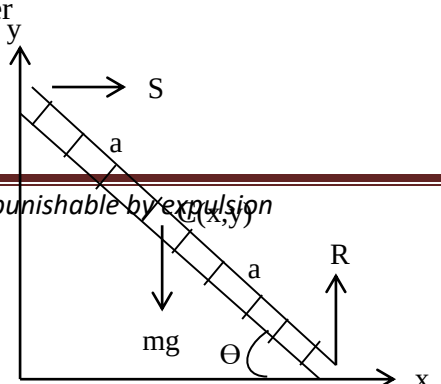

Figure 4

b) Show that the Lagrange's equation from the D'Alembert's principle is given by:

(10 marks)

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}_j}\right)-\frac{\partial L}{\partial q_j}=0$$

Where $L(q_j,\dot{q}_j,t)=T(q_j,\dot{q}_j,t)-V(q_j,t)$

QUESTION FIVE (20 MARKS)

a) The figure 5 shows a uniform cylinder. Show that the moment of inertia of the cylinder about the z-axis is given by $I_z = \frac{1}{2}MR^2$ (4 marks)

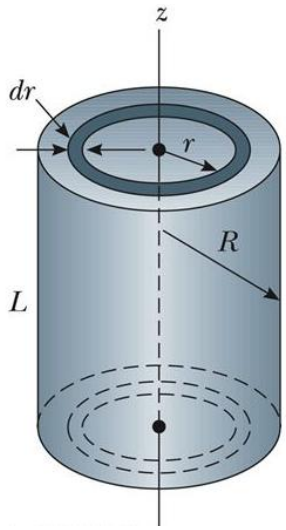

Figure 5

b) The table below show kinematic equations for rotational motion under constant acceleration. Write the corresponding kinematic equations translational motion. (6 marks)

Rotational motion about a fixed axis	Translational motion
$\omega_f = \omega_i + \alpha t$	
$\theta_f = \theta_i + \omega_i t + \frac{1}{2}\alpha t^2$	
$\omega_f^2 = \omega_i^2 + 2\alpha(\theta_f - \theta_i)$	
$\theta_f = \theta_i + \frac{1}{2}(\omega_i + \omega_f)t$	

c) A solid cylinder when dropped from a height of 2 m acquires a velocity while reaching the ground. If the same cylinder is rolled down from the top of an inclined plane to reach the ground with same velocity, Calculate the:

(i) Height h' of the inclined plane (7 marks)

(ii) Velocity of the cylinder as it touches the ground. (3 marks)

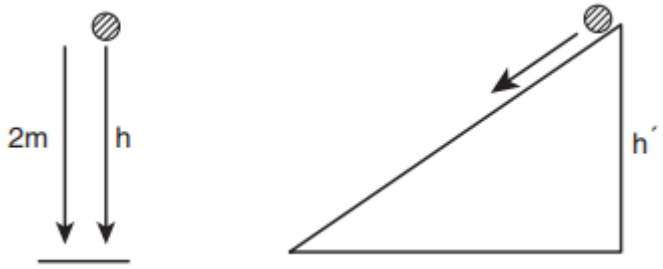

Figure 6