



MACHAKOS UNIVERSITY

University Examinations 2016/2017

SCHOOL OF PURE AND APPLIED SCIENCES

DEPARTMENT OF MATHEMATICS AND STATISTICS

FIRST YEAR SECOND SEMESTER EXAMINATION FOR DIPLOMA IN
EDUCATION

SMA 0102: INTRODUCTION TO LINEAR ALGEBRA

DATE: 2/6/2017

TIME: 2:00 – 4:00 PM

INSTRUCTIONS

Answer question ONE (Compulsory) and any other TWO questions

QUESTION ONE (COMPULSORY) (30 MARKS)

- a) Find the determinant of matrix A (6 marks)

$$A = \begin{pmatrix} 5 & 4 & 2 \\ 2 & 3 & 1 \\ 3 & -2 & -1 \end{pmatrix}$$

- b) Solve the simultaneous equation using matrix (4 marks)

$$x + 2y = 4$$

$$3x - 5y = 1$$

- c) Let $A = \begin{pmatrix} 1 & 1 \\ 1 & 2 \end{pmatrix}$ (6 marks)

i. Find the solution of $Ax = \begin{pmatrix} 3 \\ 4 \end{pmatrix}$

ii. Find the solution of $Ax = \begin{pmatrix} 3 \\ 4 \end{pmatrix}$

- d) Let $u = \begin{pmatrix} -2 \\ -9 \end{pmatrix}, v = \begin{pmatrix} 6 \\ -4 \end{pmatrix}$. Find $-2u+5v$ (4 marks)
- e) Use Cramer's rule to solve the following system of equations (6 marks)
- $$2x_1 + 3x_2 = 26$$
- $$-2x_1 + 4x_2 = 2$$
- f) By using the knowledge of determinant. Find the cross product of $u = (1,2,3)^T$ and $v = (4,5,6)^T$ (4 marks)

QUESTION TWO (20 MARKS)

- a) Let $A = \begin{pmatrix} 2 & -4 \\ -2 & -5 \\ 3 & 5 \end{pmatrix}$ and $B = \begin{pmatrix} 9 & -8 \\ -6 & -6 \\ -7 & -4 \end{pmatrix}$. Find $A+B$ (4 marks)
- b) Find the matrix product AB , if it is defined (6 marks)
- $$A = \begin{pmatrix} 0 & -2 \\ 4 & 3 \end{pmatrix} \quad B = \begin{pmatrix} -1 & 3 & 2 \\ 0 & -3 & 1 \end{pmatrix}$$
- c) Determine whether or not the matrices are inverse of each other (6 marks)
- $$\begin{pmatrix} 2 & -1 & 0 \\ -1 & 1 & -2 \\ 1 & 0 & -1 \end{pmatrix} \quad \text{and} \quad \begin{pmatrix} 1 & -1 & 2 \\ -3 & -2 & 4 \\ -1 & 1 & 1 \end{pmatrix}$$
- d) Determine the symmetric matrix corresponding to the quadratic form (4 marks)
- $$9x_1^2 + 7x_2^2 + 3x_3^2 - 2x_1x_2 + 4x_1x_3 - 6x_2x_3$$

QUESTION THREE (20 MARKS)

- a) Find the transpose of the following matrix $\begin{pmatrix} 7 & 4 & 7 & 4 \\ 0 & -7 & 0 & -7 \end{pmatrix}$ (2 marks)
- b) Find the inverse of the matrix, if it exists (4 marks)
- $$A = \begin{pmatrix} 6 & 3 \\ 3 & 0 \end{pmatrix}$$
- c) Reduced the following matrix into row echelon form. (14 marks)
- $$A = \begin{pmatrix} 2 & 3 & 1 & 5 & 2 \\ 0 & 1 & 1 & 3 & 2 \\ 4 & 5 & 1 & 7 & 2 \\ 2 & 1 & -1 & -1 & -2 \end{pmatrix}$$
- i) Find the rank of the matrix A
- ii) Find the nullity of the matrix A

QUESTION FOUR (20 MARKS)

- a) Find the inverse of matrix A (6 marks)

$$A = \begin{bmatrix} 4 & 1 & 7 \\ 2 & -3 & 1 \\ -2 & 6 & 0 \end{bmatrix}$$

- b) Determine whether the following matrix is invertible (4 marks)

$$\begin{bmatrix} 9 & 5 & -9 \\ 4 & 2 & -4 \\ -3 & 0 & 3 \end{bmatrix}$$

- c) Find the cofactors of the following matrix (6 marks)

$$\begin{bmatrix} 2 & 4 & -1 \\ 0 & 3 & 1 \\ 6 & -2 & 5 \end{bmatrix}$$

- d) For the A and B Show that $AB \neq BA$ (4 marks)

$$A = \begin{pmatrix} 2 & 3 \\ 4 & -1 \end{pmatrix} \text{ and } B = \begin{pmatrix} 5 & 1 \\ -2 & 7 \end{pmatrix}$$

QUESTION FIVE (20 MARKS)

- a) Find the eigenvalues of the given matrix (6 marks)

$$\begin{pmatrix} -14 & -6 \\ 36 & 16 \end{pmatrix}$$

- b) Diagonalize the matrix (4 marks)

$$\begin{pmatrix} 17 & -15 \\ 20 & -18 \end{pmatrix}$$

- c) The following is a system of four equations with three unknowns

$$\begin{aligned} x_1 + 2x_2 &= 2 \\ 3x_1 + 6x_2 - x_3 &= 8 \\ x_1 + 2x_2 + x_3 &= 0 \\ 2x_1 + 5x_2 - 2x_3 &= 9 \end{aligned}$$

- i. Write the augmented matrix of the equations (2 marks)

- ii. Solve the system of equations by applying the reduced echelon form (Row reduction)

(8 marks)