MACHAKOS UNIVERSITY

University Examinations 2016/2017

SCHOOL OF PURE AND APPLIED SCIENCES

DEPARTMENT OF MATHEMATICS AND STATISTICS

FOURTH YEAR SECOND SEMESTER EXAMINATION FOR DEGREE IN

BACHELOR OF EDUCATION (SCIENCE)

BACHELOR OF EDUCATION ARTS

SMA 401: TOPOLOGY II

DATE: 29/5/2017 **TIME: 2:00 – 4:00 PM**

INSTRUCTIONS TO CANDIDATES

Answer ALL the questions in Section A and ANY TWO Questions in Section B

SECTION A

QUESTION ONE (30 MARKS) COMPULSORY

a) Define the following terms
   i) Open cover
   ii) Hausdorff spaces
   iii) Compact spaces
   iv) Separated sets
   v) Locally compact sets (5 marks)

b) Prove that the open interval $A = (0,1)$ on the real line $\mathbb{R}$ with the usual topology is not compact. (5 marks)

c) Prove that every compact subset of a Housdorff space is closed. (5 marks)

d) Show that if A and B are non-empty separated sets. Then, $A \cup B$ is disconnected (5 marks)

e) Show that every metric space is hausdorff space (5 marks)

f) Prove that if F is closed subset of a compact space X, then F is also compact. (5 marks)

SECTION B

QUESTION TWO (20 MARKS)

a) Define hereditary as used in topological spaces (2 marks)

b) Prove that T_0 and T_1 spaces are hereditary (6 marks)

c) Let (X, ρ) be a topological space (X, ρ) is a T_1 space iff each singleton subset $\{x\}$ is closed in (X, ρ) (6 marks)

d) Prove that if $X, \rho)$is a topological space which is a T_2. Then every convergent sequence of points of X has a unique limit. (6 marks)

QUESTION THREE (20 MARKS)

a) Show that an infinite subset A of a discrete space X is not compact (7 marks)

b) Prove that a closed subset F of a compact set X is also compact. (7 marks)

c) Prove that a topological space X is compact if and only if $\{F_i\}$ of closed subsets of X satisfies the finite intersection property. (6 marks)

QUESTION FOUR(20 MARKS)

a) Prove that if A and B are disjoint compact subsets of Hausdorff spaces X. Then ∃ open set G and H such that $A \subset G$ and $B \subset H$ and $G \cap H = \emptyset$ (6 marks)

b) Prove that if X is compact then it is countably compact . (6 marks)

c) Prove that if X is sequentially compact then it is countably compact. (6 marks)

d) Let $\mathbb{Z}$ be the set of integers. Is it sequentially compact. (2 marks)

QUESTION FIVE (20 MARKS)

a) Consider the following topology on $X = \{a, b, c, d, e\}$ $\rho = \{X, \emptyset, \{a, b, c\}, \{c, d, e\}, \{c\}\}$. Now $A = \{a, d, e\}$. Show that A is disconnected. (5 marks)

b) Let $G \cap H$ be a disconnection of A. show that $G \cap A$ and $H \cap A$ are separated. (5 marks)

c) Prove that a set is disconnected if and only if it is not a union of two non-empty separated sets. (5 marks)

d) Let $G \cup H$ be a disconnection of A and B be a connected subset of A. Show that $B \cap G = \emptyset$ or $B \cap H = \emptyset$. (5 marks)