MACHAKOS UNIVERSITY

University Examinations 2016/2017

SCHOOL OF PURE AND APPLIED SCIENCES

DEPARTMENT OF MATHEMATICS AND STATISTICS

FIRST YEAR SECOND SEMESTER EXAMINATION FOR DEGREE IN

BACHELOR OF SCIENCE IN STATISTICS AND PROGRAMMING

SST 103: LINEAR ALGEBRA

DATE: 5/6/2017 **TIME: 2:00 – 4:00 PM**

INSTRUCTIONS TO CANDIDATES

Answer ALL the questions in Section A and ANY TWO Questions in Section B

SECTION A(COMPULSORY)

QUESTION ONE (30 MARKS)

a) Find the determinant of the following matrix

$$A = \begin{pmatrix} 2 & 3 & 6 \\ 1 & 1 & 2 \\ 2 & 3 & 4 \end{pmatrix}$$ (3 marks)

b) Reduce the following matrix to be reduced echelon form

$$A = \begin{pmatrix} 3 & 4 & -1 & 1 \\ 1 & -1 & 3 & 2 \\ 4 & -3 & 11 & 2 \end{pmatrix}$$ (3 marks)

c) Solve the following simultaneous equation using inverse matrix method

$2x - y = 4$

$3x + 2y = 6$ (3 marks)

d) If $\begin{vmatrix} y & 5 \\ y & 2y+1 \end{vmatrix} = 4$ find y (3 marks)

e) Calculate the cross product of the vectors $\vec{u}$ = (1, 2, 3) and $\vec{v}$ = (−1, 1, 2). (3 marks)

f) Determine the inverse of the following matrix

$$\begin{bmatrix} 0 & 2 & 4 \\ 2 & 4 & 2 \\ 3 & 3 & 1 \end{bmatrix}$$

(4 marks)

g) Determine the value of x and y for the vector (x, y, 1) that is orthogonal to the vectors (3, 2, 0) and (2, 1, −1). (4 marks)

h) Determine if (1,2,3,1), (2,2,1,2) and (-1,2,7,3) is linearly dependent (4 marks)

i) Determine the angle between $u = 2i$ +2j+2k and $v = i + j + k$ (3 marks)

SECTION B: ANSWER ANY OTHER TWO QUESTIONS

QUESTION TWO (20 MARKS)

a) Find the rank of the following matrix

$$\begin{bmatrix} 2 & 4 & 3 \\ 3 & -3 & 2 \\ 2 & 0 & 3 \end{bmatrix}$$

(4 marks)

b) Transpose the following matrix

$$\begin{pmatrix} 1 & 2 & 3 \\ 2 & 4 & 6 \\ 3 & 4 & 7 \end{pmatrix}$$

(2 marks)

c) Find the minors, cofactors and adjoint of the following matrix

$$\begin{bmatrix} 2 & 3 & 1 \\ 2 & 3 & 2 \\ 1 & 3 & 1 \end{bmatrix}$$

(4 marks)

d) Find the inverse of the following matrices

$$\begin{pmatrix} \cos\theta & \sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

(5 marks)

e) Reduce the following matrix into echelon form

$$\begin{pmatrix} 1 & 0 & 1 & 1 \\ 0 & 1 & 1 & 2 \\ 2 & 1 & 1 & 0 \end{pmatrix}$$

(5 marks)

QUESTION THREE (20 MARKS)

a) Solve by Crammer's Rule

$$x - 2y + 3z = 10$$

$$3x - 6y + z = 22$$

$$-2x + 5y - 2z = -18$$

(5 marks)

b) Determine the value of 'a' so that the following systems in unknown *x*, *y* and *z* has

i). No solution

ii). More than one solution

iii). Unique solution

$$x - 3z = -3$$
$$2x + ay - z = -2$$
$$x + 2y + az = 1$$

(6 marks)

c) Solve the following system of equation by elimination method

$$x_1 + x_2 + x_3 = 5$$

$$x_1 + 2x_2 + 3x_3 = 10$$

$$2x_1 + x_2 + x_3 = 6$$

(4 marks)

d) Solve the simultaneous equation using the gauss-Elimination method

$$2x_1 - 4x_2 + 6x_3 = 20$$

$$6x_1 - 12x_2 + 2x_3 = 44$$

$$-4x_1 + 10x_2 - 4x_3 = -36$$

(5 marks)

QUESTION FOUR (20 MARKS)

a) Show that *u*, v, and *w* are linearly independent

$u = (6, 2, 3, 4)$

$v = (0, 5, -3, 1)$

$w = (0, 0, 7, -2)$ (5 marks)

b) Write $E = \begin{pmatrix} 3 & 1 \\ 1 & -1 \end{pmatrix}$ as a linear combination of the matrices

$A = \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix}$, $B = \begin{pmatrix} 0 & 0 \\ 1 & 1 \end{pmatrix}$ and $C = \begin{pmatrix} 0 & 2 \\ 0 & -1 \end{pmatrix}$ (5 marks)

c) Calculate the value of k for the vectors $\vec{u}$ = (1, k) and $\vec{v}$ = (−4, k) knowing that they are orthogonal. (5 marks)

d) Show that the vectors u=(1,-1,0) v=(1,3,-1) and w=(5,3,-2) are linearly dependent. (5 marks)

QUESTION FIVE (20 MARKS)

a) Define a vector space (6 marks)

b) Show that W= $\{(x, y) \;/\; x = 2y\}$ is a subspace for R^2. (4 marks)

c) Prove that the diagonals of a rhombus are perpendicular. 5 marks)

d) Find the parametric and the symmetric equations of the line passing through the point $(2, 3, -4)$ and parallel to the vector $(3,5,-6)$ (5 marks)