MACHAKOS UNIVERSITY

University Examinations 2016/2017

SCHOOL OF PURE AND APPLIED SCIENCES

DEPARTMENT OF MATHEMATICS AND STATISTICS

FOUTH YEAR SECOND SEMESTER EXAMINATION FOR DEGREE IN BACHELOR OF EDUCATION (SCIENCE)
BACHELOR OF EEDUCATION (ARTS)

SMA 433: PARTIAL DIFFERENTIAL EQUATIONS II

DATE: 5/6/2017 **TIME: 2:00 – 4:00 PM**

INSTRUCTION TO CANDIDATES:

Answer Question One and Any Two Other Questions.

QUESTION ONE (COMPULSORY) (30MARKS)

a) Evaluate the solution to the following initial value problem
$u_{xx} = 4xy + e^x$
with initial conditions
$u(0, y) = y,\ u_x(0, y) = 1$ (5 marks)

b) Solve the equation $\frac{\partial^4 z}{\partial x^4} + \frac{\partial^4 z}{\partial y^4} = 2\frac{\partial^4 z}{\partial x^2 \partial y^2}$ (5 marks)

c) Determine the particular integral of the partial differential equation
$$\frac{\partial^2 z}{\partial x^2} - \frac{\partial z}{\partial y} = 4e^{2x+4y}$$ (6 marks)

d) Consider the equation
$u_{xx} + 4u_{xy} + 4u_{yy} = 0$. Classify the equation and find the general solution. (6 marks)

e) Find the Fourier transform of $f(x) = e^{-ax^2}$; $a > 0$ (8 marks)

QUESTION TWO (20 MARKS)

a) Obtain a partial differential equation of the first order by eliminating the arbitrary function
$u = e^{-y} f(x) + e^x g(y)$ (4 marks)

b) Reduce the equation $r - x^2 t = 0$ to canonical form (8 marks)

c) Show that Laplace's equation in three dimensions

$$u_{xx} + u_{yy} + u_{zz} = 0$$ (8 marks)

QUESTION THREE (20 MARKS)

a) i) Determine the characteristics of equation $r - yt = 0$

ii) Show under which condition the curve will be a characteristic of the partial differential equation $p^2 r + 3pqs + 2q^2 t = 0$ (5 marks)

b) Find the complete solution of the partial differential equation

$$\frac{\partial^2 z}{\partial x^2} - \frac{\partial^2 z}{\partial y^2} = (x - y)$$ (5 marks)

c) Reduce the equation $r + 2s + t = 0$ into canonical form and solve completely the corresponding equation. (10 marks)

QUESTION FOUR (20 MARKS)

a) An infinitely long string having one end at $x = 0$ is initially at rest on the $x-$axis. At $t = 0$ the end $x = 0$ begins to move along the x-axis in a manner described by $u(0,t) = a\cos\sigma t$. Find the displacement $u(x,t)$ of the string at any point at any time.

(10 marks)

b) Determine the general solution of the partial differential equation

$$x^2 z_{xx} - 2xyz_{xy} - y^2 z_{yy} - xz_x + yz_y - 5z = 7x - 5y$$ (10 marks)

QUESTION FIVE (20 MARKS)

Given the partial differential equation

$r - t = 0$ …… (i)

and five real functions $\left.\begin{aligned} x &= t \\ y &= 0 \\ z &= t^2 + t^4 \end{aligned}\right\}$ ……(ii) and

$\left.\begin{aligned} p &= 2t + 4t^3 \\ q &= 4t \end{aligned}\right\}$ ……(iii).

Show that the five real fuctions form a strip hence interpret geometrically initial value problem in which we seek a solution of the form $z = z(x, y)$ (20 marks)