



MACHAKOS UNIVERSITY

University Examinations 2016/2017

SCHOOL OF PURE AND APPLIED SCIENCES

DEPARTMENT OF MATHEMATICS AND STATISTICS

SECOND YEAR SECOND SEMESTER EXAMINATION FOR

DIPLOMA IN ELECTRICAL AND ELECTRONICS ENGINEERING

DIPLOMA IN BUILDING AND CIVIL ENGINEERING

DIPLOMA IN MECHANICAL ENGINEERING

CALCULUS III/MATHEMATICS VI

DATE: 30/5/2017

TIME:8:30 – 10:30 AM

INSTRUCTION TO CANDIDATES:

Answer Questions One and Any Other Two Questions

QUESTION ONE (COMPULSARY 30 MARKS)

a) Define the following terms

- i) iteration (2 marks)
- ii) iterative

b) Derive the Newton Raphsons formula

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)} \quad (5 \text{ marks})$$

c) Use the maclaurins series to find the series for the following functions

- i) $\cos x$ (3 marks)
- ii) $\ln x$ (2 marks)

d) i) Find the maclaurins series for the function

$f(x) = \ln \frac{1+x}{1-x}$ hence evaluate $\int_0^1 \ln \frac{(1+x)}{(1-x)} dx$ using the first four terms of the series (10 marks)

- e) Obtain the Taylor's series for the following functions
- i) $\ln(1+x)$ (2 marks)
- ii) e^x (2 marks)
- f) Given that $\sin 30^\circ = 0.5$ Determine the value of $\cos 40^\circ$ by Taylor's series (4 marks)

QUESTION TWO (20 MARKS)

- a) Determine the Fourier series for the periodic function $f(t)$ where

$$f(t) = \begin{cases} t & 0 \leq t < 2\pi \\ f(t + 2\pi) & \end{cases} \quad (13 \text{ marks})$$

- b) Determine the half range Fourier sine series for the periodic function

$$f(x) = \begin{cases} 1+x & 0 < x < 5 \\ f(x+10) & \end{cases} \quad (7 \text{ marks})$$

QUESTION THREE (20 MARKS)

- a) Given that $\cos 30^\circ = 0.8660$ determine the value of $\cos 40^\circ$ by Taylor's series (5 marks)
- b) Determine the value of $\int_0^1 \frac{\cos 2x}{x^{1/3}} dx$ correct to 2 decimal places. (5 marks)
- c) i) Derive Fourier series coefficients for half range sine series with a period T . (5 marks)

- ii) Given $f(x) = \begin{cases} 3t & 0 < t < 1 \\ 3 & 1 < t < 2 \\ f(t+2) & \end{cases}$
 find the Fourier series expansion. (5 marks)

QUESTION FOUR (20 MARKS)

- a) i) Given the polynomial $f(x) = 4x^3 - 6x^2 + 15x + 4$ prove that the Newton-Raphson's interpolation formulae is given by $x_{n+1} = \frac{8x^3 - 6x^2 + 4}{12x^2 - 12x + 15}$ (5 marks)
- ii) Taking $x_0 = 0.7$ obtain a better approximation to the root of the equation correct to six decimal places. (5 marks)

b) Given the table

x	-4	-2	0	2	4	6	8
F(x)	-44	6	8	10	60	206	496

- i) Construct a finite table of differences (3 marks)
- ii) Use the table to obtain the values of $f(-3.7)$, $f(6.5)$ correct to three decimal places (7 marks)

QUESTION FIVE (20 MARKS)

Given $x_n = 1.234$ $x_{n+1} = 1.2447$

$$x_{n+2} = 1.3124 \quad x_{n+3} = 1.3233$$

$$f(x_n) = 12.5674 \quad f(x_{n+2}) = 13.9831$$

Use linear interpolation and extrapolation to calculate $f(x_{n+1})$ and $f(x_{n+3})$

To 5 decimal places. (20 marks)