



# MACHAKOS UNIVERSITY

University Examinations 2017/2018

SCHOOL OF PURE AND APPLIED SCIENCES

DEPARTMENT OF MATHEMATICS AND STATISTICS

SUPPLEMENTARY EXAMINATION FOR

DIPLOMA IN ELECTRICAL AND ELECTRONICS ENGINEERING

DIPLOMA IN BUILDING AND CIVIL ENGINEERING

DIPLOMA IN MECHANICAL ENGINEERING

APPLIED GEOMETRY

DATE:

TIME:

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## INSTRUCTIONS

Attempt question ONE and any other two questions

### QUESTION ONE (COMPULSORY) (30mrks)

- I. A triangular template has sides of 8cm, 7cm and 5cm. Find its three angles  
(3marks)
- II. Use the Cosine rule to solve the following triangle XYZ.  
 $y=11\text{cm}$ ,  $z=15\text{cm}$ ,  $X=55^\circ$   
(3marks)
- III. Two ships, A and B, leave a port at the same time. A sails at a steady speed of 52km/h,  $S32^\circ W$  and B at 38km/h  $S24^\circ E$ . Find their distance apart after 2.5hours.

(3marks)

IV. Find the volume and the total surface area of a cone of radius 4.86cm and perpendicular height 10.58cm. (3marks)

V. A hemisphere has a diameter of 3.6cm. Find its volume and total surface area.

(3marks)

VI. A cone of height  $d$  is cut by a plane parallel to a distance  $\frac{d}{3}$  from the base. Show that, the volume of the frustum produced is 70.37% of the original cone. (3marks)

VII. Prove the following trigonometric identities

a.)  $\sin \theta \cos \theta = \frac{\sin^2 \theta}{\tan \theta}$

(3marks)

b.)  $\sin \theta \sec \theta = \tan \theta$

(3marks)

VIII. Verify the following identity

$$\sin \theta + \sin \phi = 2 \sin \frac{\theta + \phi}{2} \cos \frac{\theta - \phi}{2}$$

(4marks)

IX. If  $\mathbf{a} = 2\mathbf{i} + 2\mathbf{j} - \mathbf{k}$  and  $\mathbf{b} = 3\mathbf{i} - 6\mathbf{j} + 2\mathbf{k}$ , find  $\mathbf{a} \cdot \mathbf{b}$  and  $\mathbf{a} \times \mathbf{b}$  (2marks)

### QUESTION TWO (20MARKS)

I. What is the length of a diagonal of a square of side length  $\sqrt{2}$  (2marks)

II. If  $\tan \theta = a$ , show that;

$$\frac{\cos \theta \sin^2 \theta + \cos^3 \theta}{\sin \theta} = \frac{1}{a}$$

(3marks)

III. A regular hexagonal pyramid has a perpendicular height of 42mm. if the distance across the flats of the hexagonal base is 18mm find the volume and the lateral surface area of the pyramid. (lateral surface area means the area of the sides of the figure)

(5marks)

IV. Find the volume and the total surface area of a cone of radius 4.86cm and perpendicular height 10.58cm

(5marks)

V. A cone 15cm high and of base diameter 12cm is cut by a plane parallel to the base and 9cm from the base. Find the ratio of the volume of the two parts thus formed.

(5marks)

**QUESTION THREE (20MARKS)**

- I. Show that the points  $A = (1,3,5)$ ,  $B = (4,12,20)$  and  $C = (3,9,15)$  are collinear. (3 marks)
- II. In triangle  $PQR$ ,  $M$  is the midpoint of  $PR$ .  $N$  is a point on  $PQ$  such that  $3PN = 2NQ$ .  $NM$  produced meets  $QR$  produced at  $L$ . Determine the ratios in which  $L$  divides  $QR$ . (3 marks)
- III. Use the fundamental trigonometric identity to show that:  
a.) The triangle with sides 5cm, 12cm and 13cm is a right-angled triangle (3 marks)  
b.) The triangle with sides 7cm, 15cm and 16cm is a right-angled triangle (3 marks)
- IV. Verify the following trigonometric identity:  
a.)  $\frac{(\cos \theta - \sin \theta)^2}{\cos \theta} = \sec \theta - 2 \sin \theta$  (3 marks)
- V. Find the volume and total surface area of a frustum of a pyramid, the ends being squares of sides 6.4m and 3.6m and the height being 4m. (5 marks)

**QUESTION FOUR (20MARKS)**

- I. Show that  
a.)  $\tan 75^\circ = \frac{\sqrt{3} + 1}{\sqrt{3} - 1}$  (3 marks)  
b.)  $\sin 15^\circ = \frac{\sqrt{3} - 1}{2\sqrt{2}}$  (3 marks)
- II. Find non-zero scalars  $\mu$  and  $\beta$  such that  $\mu\mathbf{a} + \beta\mathbf{b} = \mathbf{c}$ , given that  $\mathbf{a} = \begin{pmatrix} 3 \\ 2 \end{pmatrix}$ ,  $\mathbf{b} = \begin{pmatrix} 4 \\ 6 \end{pmatrix}$ ,  $\mathbf{c} = \begin{pmatrix} 2 \\ 2 \end{pmatrix}$  (3 marks)
- III. Given that  $\mathbf{r} = 2\mathbf{i} + 3\mathbf{j} + 4\mathbf{k}$ ,  $\mathbf{s} = 4\mathbf{i} - \mathbf{j} + 5\mathbf{k}$  and  $\mathbf{t} = -\mathbf{i} + 9\mathbf{j} + 5\mathbf{k}$ , express  $\mathbf{r}$  as a linear combination of  $\mathbf{s}$  and  $\mathbf{t}$ . (3 marks)
- IV. A cooling tower is in the form of a cylinder of height 15m surmounted by a frustum of a cone. The diameter of the cylinder and the bottom of the frustum is 30m and

the diameter at the top of the tower is 18m. The height of the tower is 40m. Calculate the volume of the air space in the tower if 35% of the space is used for pipes and other structures. (5marks)

- V. A triangular pyramid has a perpendicular height of 25cm. If the base is an equilateral triangle of side 3cm, find its volume. (3marks)

**QUESTION FIVE (20MARKS)**

- I. Proof  $\sin (a+b)=\sin a \cos b + \cos a \sin b$  for angle  $(a+b)<90^\circ$  (10marks)

- II. Solve the equation  $3\cos 2\theta + \sin \theta = 1$  for values of  $\theta$  from  $0^\circ$  to  $360^\circ$  inclusive

(5marks)

- III. Prove that  $\sin 3A = 3\sin A - 4\sin^3 A$  (5marks)