MACHAKOS UNIVERSITY

University Examinations 2016/2017

SCHOOL OF PURE AND APPLIED SCIENCES

DEPARTMENT OF MATHEMATICS AND STATISTICS

THIRD YEAR FIRST SEMESTER EXAMINATION FOR

BACHELOR OF SCIENCE (MATHEMATICS)

SMA 362: OPERATIONS RESEARCH 1

DATE: 31/7/2017 **TIME: 11:00 – 1:00 PM**

INSTRUCTIONS

1. ***Answer Question* 1 *and any other two questions.***
2. ***You must have the following items for this paper:***
   - ***Scientific Calculator;***
   - ***Graph paper.***

1. (a) (i) Outline **four** conditions which must be satisfied for a problem to be solved using linear programming technique. (4 marks)

   (ii) Outline any **four** sources of constraints in an optimisation problem involving linear programming. (2 marks)

   (b) Explain each of the following types of variables as used in linear programming:

   (i) Slack variable;

   (ii) Surplus variable. (4 marks)

   (c) A factory produces four types of electrical cables – A, B, C and D. The factory wishes to maximise contribution to revenue. The sale prices per metre of the cables are 40, 32, 60 and 54 Kenya shillings respectively. The factory employs 200 skilled workers, 150 unskilled workers, and 100 casual workers who work a 40 hour week. The time it takes to produce one metre of each type of cable is as shown in the table below:

Product		A	B	C	D
Hours	Skilled	5	3	2	6
	Unskilled	4	7	3	5
	Casual	7	6	4	8

   Formulate a linear programming model for this problem. (6 marks)

(d) Given the linear programming model below:

$$
\begin{aligned}
\text{Minimise :} \quad & z = 8x_1 + 12x_2 + 5x_3 \\
\text{Subject to :} \quad & 2x_1 + 3x_2 + 6x_3 \geq 20 \\
& 6x_1 + 8x_2 + 5x_3 \geq 40 \\
& 7x_1 + 3x_2 + 6x_3 \geq 50 \\
\text{With :} \quad & x_1, \; x_2, \; x_3 \geq 0
\end{aligned}
$$

Determine its symmetrical dual program. (2 marks)

(e) A flour milling plant produces two types of flour - wheat and maize. Wheat flour is sold at Ksh 80 per packet while maize flour is sold at Ksh 70 per packet. The manager of the firm wants to determine the daily production plan which maximises sales. The production constraints are as spelt out in the table below:

	Labour hours	Machine hours	Packaging Material
Wheat	2	2	1
Maize	3	1	1
Maximum available	180	120	80

A hotel which is a customer to the flour mill has placed a standing order for the supply of a minimum of 20 packets of wheat flour per day.

(i) Formulate a linear programming model for the problem. (3 marks)

(ii) Using the graphical method, determine the optimum daily production plan which maximises sales for the flour firm. (6 marks)

(iii) Interpret the optimum solution obtained in (ii). (3 marks)

2. A dairy production plant produces two products - cheese and milk. Cheese costs Ksh 60 per unit while milk costs Ksh 45 per unit. The manager of the firm wants to determine the daily production plan which minimises the cost of production. The production constraints are as spelt out in the table below:

	Labour cost	Machine cost	Packaging material
Cheese	2	2	1
Milk	3	1	1
Minimum cost	180	120	80

The Ministry of Health has issued a directive that a minimum of 20 units of cheese must be produced per day in conformity with a health strategy.

(i) Formulate a linear programming model for the problem. (4 marks)

(ii) Using the graphical method, determine the optimum daily production plan which minimises the cost of production for the dairy plant. (12 marks)

(iii) Interpret the optimum solution obtained in (ii) above. (4 marks)

3. A fruit farm which produces three types of fruit juices – mango, orange and pineapple, has its weekly production plan modelled as an LP program as shown below.

$$\text{Maximise}: \quad z = 80x_1 + 50x_2 + 100x_3$$

$$\begin{aligned}
\text{Subject to}: \quad 2x_1 + 3x_2 + x_3 &\leq 4000 \\
x_1 + x_3 &\leq 1500 \\
2x_1 + 4x_3 &\leq 2000 \\
x_2 &\leq 500
\end{aligned}$$

$$\text{With}: \quad x_1, \; x_2, \; x_3 \geq 0$$

(a) Using the Simplex method, determine the optimum weekly production plan for the farm. (16 marks)

(b) Interpret the solution obtained in (a) above. (4 marks)

4. Below is a linear programming model. Use it to answer the questions that follow.

$$\text{Minimise}: \quad z = 50x_1 + 80x_2 + 90x_3$$

$$\begin{aligned}
\text{Subject to}: \quad x_1 + x_2 + 2x_3 &\geq 30 \\
x_1 + 2x_2 + x_2 &\geq 40
\end{aligned}$$

$$\text{With}: \quad x_1, \; x_2, \; x_3 \geq 0$$

(a) Determine the symmetrical dual of the LP programming model above. (4 marks)

(b) Using the Simplex method, determine the optimum solution for the linear program problem by solving the dual program. (10 marks)

(c) Using the relationship between a primal program and its symmetrical dual program, derive the optimum solution of the primal program from the optimum solution of its dual. (6 marks)

5. An oil distribution company is required to meet the demands 50, 70, 100, 60 in thousand litres at various destinations from supplies 100, 60, 120 in thousand litres from various sources. The transport cost per unit amount (thousand litres) over the various routes are as given in the matrix below:

$$C = \begin{bmatrix} 5 & 1 & 3 & 4 \\ 2 & 4 & 2 & 5 \\ 1 & 3 & 2 & 4 \end{bmatrix}$$

The company wants to meet the demand at destinations by transporting the oil at the cheapest cost possible. Using the northwest corner (NWC) rule, do the following for the transportation problem:

(a) Derive the initial basic solution for the problem; (3 marks)

(b) Determine the optimal solution that minimizes the cost of transport; (14 marks)

(c) Interpret the optimum solution obtained in (b) above. (3 marks)