Journal of Physics: Conference Series

Statistical techniques for modeling extreme price dynamics in the energy market

To cite this article: L N Mbugua and P N Mwita 2013 *J. Phys.: Conf. Ser.* **410** 012113

Statistical techniques for modeling extreme price dynamics in the energy market

L N Mbugua[1] and P N Mwita[1]

[1]Jomo Kenyatta University of Agriculture and Technology, Department of Statistics and Actuarial Sciences, P.O Box 62000-00200, Nairobi, Kenya

Email: mbugualevi@yahoo.co.uk

Key words: Extreme value theory, Peak over threshold, Quantile regression, Value at risk

Abstract. Extreme events have large impact throughout the span of engineering, science and economics. This is because extreme events often lead to failure and losses due to the nature unobservable of extra ordinary occurrences. In this context this paper focuses on appropriate statistical methods relating to a combination of quantile regression approach and extreme value theory to model the excesses. This plays a vital role in risk management. Locally, nonparametric quantile regression is used, a method that is flexible and best suited when one knows little about the functional forms of the object being estimated. The conditions are derived in order to estimate the extreme value distribution function. The threshold model of extreme values is used to circumvent the lack of adequate observation problem at the tail of the distribution function. The application of a selection of these techniques is demonstrated on the volatile fuel market. The results indicate that the method used can extract maximum possible reliable information from the data. The key attraction of this method is that it offers a set of ready made approaches to the most difficult problem of risk modeling

1. Introduction

The extreme rate of change of production factors and their corresponding factor payments expose both the producers and consumers to significant risks, which they can hedge provided that they have good modeling tools of which they can forecast these changes well enough [1]. One major determinant that affects production capacity and the market mechanism structure is the cost of energy [2]. There exists a wide range of mathematical approaches to modeling and forecasting costs as well as price dynamics, but often do perform poorly in case of extreme events [3]. In this paper we focus on regression quantiles to capture the complete underlying structure of a time series data and model the observations that exceed central limit, focusing on the tail of the distribution. We apply Extreme Value Theory (EVT) to capture extremism [4]. Value at risk, the minimal loss which may occur under extra ordinary market circumstances is also discussed.

2. Empirical quantiles

When data has non-constant mean and variance, typically skewed and contains some outliers, it creates difficulties in empirical modelling [5]. More of interest is where the data pattern shows heteroscedasticity and asymmetries. In such data, quantile regression an order statistic is more explicable and robust than the mean regression [6].

2.1 Definition

The q[th] quantile estimate of the distribution function F that is $\hat{F}_k^{-1}(q)$ is defined as

$$(q) = inf\{x|\hat{F}_k(x) \geq q\},\ 0 < q < 1 \tag{1}$$

where, $\hat{F}_k$ is the empirical distribution function that put mass $1/k$ at each $X_i, 1 \leq i \leq k$ that is

$$\hat{F}_k(x) = \frac{1}{k}\sum_{i=1}^{k} I_{\{X_i \leq x\}},\ \ -\infty < x < \infty$$

with I being the indicator function and inf denotes the smallest real number x satisfying $F_k(x) \geq q$. If a random variable $K(k)$ of $F_k(x)$ is continuous, then $q = Pr[K(k) \leq F_k^{-1}]$.

Applying the Rosenblatt-Parzen kernel estimators of the density function f we get

$$\hat{f}(x) = \frac{1}{k}\sum_{i=1}^{k}\frac{1}{h}w\left(\frac{x-X_i}{h}\right),\ -\infty < x < \infty \tag{2}$$

where w is a kernel which is non negative and that $\int_{-\infty}^{\infty} w(x)dx = 1$. h, is a positive integer which governs the smoothness of the fluctuations. A relatively large value of h gives too much smoothness and a relatively small value of h give big fluctuations. Consequently, with $W(x) = \int_{-\infty}^{x} w(y)dy$ the distribution function corresponding to the density w is given by

$$\hat{F}_W(x) = \frac{1}{k}\sum_{i=1}^{k} W\left(\frac{x-X_i}{h}\right),\ \ -\infty < x < \infty \tag{3}$$

which is a kernel estimator of the distribution function F. A smooth alternative to equation (1) may be defined by

$$\hat{F}_W^{-1}(q) = inf\left\{x\left|\frac{1}{k}\sum_{i=1}^{k} W\left(\frac{x-X_i}{h}\right) \geq q\right.\right\},\ \ 0 < q < 1\,. \tag{4}$$

Let the observations Y_t be as a realization of the model;

$$Y_t = \hat{q}_\theta^t + \hat{\epsilon}_\theta^t \tag{5}$$

where $\hat{q}_\theta^t$ is an estimate of the quantile defined in equation (4) and $\hat{\epsilon}_\theta^t = Y_t - \hat{q}_\theta^t$ are the standardized quantile residuals. It turns out that

$$\frac{\hat{\epsilon}_\theta^t}{\hat{q}_\theta^t} = \frac{Y_t - \hat{q}_\theta^t}{\hat{q}_\theta^t} = \frac{Y_t}{\hat{q}_\theta^t} - 1 \tag{6}$$

θ, corresponds to a reasonably low probability for which the quantile can be estimated nonparametrically. Let p denote a very small probability of interest, reformulating the definition of the $p-th$ of the variable rate of change in terms of the $\theta - th$ quantile yields

$$P[Y_t < q_p^t] = P[Y_t < q_\theta^t - q_\theta^t + q_p^t] \tag{7}$$

Assuming that q_p^t is a negative number, we switch the inequality sign to obtain

$$P\left[\frac{Y_t}{q_\theta^t} - 1 > \frac{q_p^t}{q_\theta^t} - 1\right] = \theta \tag{8}$$

Let $\frac{q_p^t}{q_\theta^t} - 1 \equiv z_p$ denote the $p-th$ quantile of the standardized residuals. Then

$$\hat{q}_p^t = (\hat{z}_p + 1)\hat{q}_\theta^t\,. \tag{9}$$

Since data sparseness is more severe in extreme quantiles, we embark upon refining nonparametric quantile regression methods with extreme value theory so as to be proficient to model extreme quantiles accurately. To obtain $\hat{z}_p$ in equation (9) we apply the idea of the standard peaks over threshold (POT) methods [7].

3. Modeling Extreme Changes

The distribution function of the excesses over the threshold u can be defined by

$$F_u(x) = Pr\{X - u \leq x|X > u\} = \frac{F(x+u)-F(u)}{1-F(u)} \tag{10}$$

Theorem: For a large class of underlying distribution function F, the conditional excess distribution function $F_u(x)$, for u large is well approximated by Pickands and Balkema [8], that is

$$\lim_{u\to x_0} \sup_{0\le x\le x_0-\mu}\left|F_u(x) - H_{\xi,\sigma}(x)\right| = 0 \tag{11}$$

where

$$H_{\xi,\sigma}(x) = \begin{cases} 1-\left(1+\xi\frac{x}{\sigma}\right)^{-1/\xi} & \xi \neq 0 \\ 1-e^{(-x/\sigma)} & \xi = 0 \end{cases} \tag{12}$$

ξ is the shape parameter and $\frac{1}{\xi}$ is the tail index. The tail index sets the rate at which the tails of $H_{\xi,\sigma}(x)$ decay away. $\sigma > 0$, is the scale parameter. The tail index as well as the scaling parameter has to be determined by fitting equation (12) to the actual data and estimate the parameters with the maximum likelihood method.

4. Peak over a high threshold (POT)

Denote the random variable of interest by r_t. Let u be a specified high threshold. Suppose that the $i-th$ exceedance at time t_i is $r_{ti} \le u$, we shall focus on the data $(t_i, r_{ti} - u)$ where $r_{ti} - u$, are the excesses over the threshold u. The occurrence times $\{t_i\}$ provides useful information about the intensity of the occurrence of important "rare events". A cluster of t_i indicates a period of large declines and the exceeding amount provides the actual quantity of interest. Different choices of the threshold u leads to different estimates of the tail index $\frac{1}{\xi}$ and is based on risk tolerance. Since different institutions and investors have different risk tolerance, the choice of u is a statistical problem as well as an institutional one. Too low u value and the asymptotic theory breaks down, too high threshold value and one does not have enough data points to estimate the parameters [9]. Rearranging equation (10) and using $F_u(\cdot) \approx H_{\xi,\sigma}(r_t)$, it holds that

$$1-F(r_t+u) \approx [1-F(u)][1-H_{\xi,\sigma}(r_t)]$$

We estimate $1-F(u) = z(u)$ by use of empirical distribution function and

$$z(u+r_t) = \frac{N_u}{n}\left(1+\hat{\xi}\left(\frac{r_t}{\hat{\beta}}\right)\right)^{-\frac{1}{\hat{\xi}}} \tag{13}$$

where, N_u denotes the number of excesses over the threshold u. Employing the change in variables $y = u + r_t$ and fixing the distribution value at the probability of interest, $F(y) = \theta$, we estimate the quantile estimator $\hat{q}_\theta$ by inverting equation (13) to obtain

$$1-\theta = \frac{N_u}{n}\left(1+\hat{\xi}\left(\frac{y-u}{\hat{\beta}}\right)\right)^{-\frac{1}{\hat{\xi}}}.$$

$$\hat{q}_\theta = u + \frac{\hat{\beta}}{\hat{\xi}}\left[\left((1-\theta)\frac{n}{N_u}\right)^{-\hat{\xi}} - 1\right] \tag{14}$$

Thus the θ^{th} quantile value at risk of $F(y)$ can be estimated by

$$VaR_\theta = u - \frac{\beta}{\xi}\left[1-\left[\frac{n}{N_u}(1-\theta)\right]^{-\xi}\right] \tag{15}$$

Equation (15) is an analytical tool which can be used for assessing riskiness of trading activities therein, Value at risk (VaR). Consequently, the conditional expectation of loss given that the loss is beyond VaR level is the expected shortfall.

5. Results and discussion

We focus on the distribution of daily gasoline price changes in Kenya from July 1, 2006 to June 20, 2009 (data from Kenya National Oil Corporation). Since the price changes are so high we chose logarithmic changes instead of simple net returns. A problem of using simple net returns is that prices are bounded from below and that this makes the return distribution skewed for large positive and

negative returns. We focus only on the negative tail of the distribution even though the same concept applies to positive tails.
Table 1, reports some statistics on the price changes series. The high volatility is confirmed as evidenced by the very high excess kurtosis. The very small P-value of the Pormanteau test (Ljung) together with visual inspection of figure 1 indicates a high degree of volatility clustering effect.

Table 1 Descriptive statistics of daily gasoil price changes in Kenya.

Statistic	Value
Mean	-7.958×10^{-5}
Standard deviation	0.209
Skewness	0.289
Kurtosis	23.379
Ljung P-Value	2.20×10^{-16}

To get better tail estimates, we first filter the data to capture some of the most important dependencies and thereafter apply the ordinary extreme value technique. The advantage is that independent and identical assumption behind the EVT based tail quantile estimator is less likely to be violated.
To pre filter the time series we combine the Autoregressive moving average (ARMA) with the simplest generalized autoregressive conditional heteroscedastic (GARCH (1, 1)) model. Thus we have the following model

$$r_t = a_0 + a_1 r_{t-1} - b_1 e_{t-1} + e_t.$$

$$\sigma_t^2 = \varphi_0 + \varphi_1 e_{t-1}^2 + \varphi_2 \sigma_{t-1}^2 \quad (16)$$

where σ_t^2 is the conditional variance of e_t and $e_t = \sigma_t \varepsilon_t$ with $\varepsilon_t \sim N(0,1)$. Let the unconditional EVT quantiles of the residual distribution be $\hat{q}_\theta$, the conditional tail quantiles of our original rate of change distribution will be given by

$$q_{t,\theta} = a_0 + a_1 r_{t-1} - b_1 e_{t-1} + \sigma_t \hat{q}_\theta \quad (17)$$

In implementing POT method, we rely on a reasonable choice of the threshold that is approximately 5.5%. We produce a one step a head forecast of the conditional mean volatility using equation (16) and use these values to scale up our residual quantiles with equation (17). The summary statistics are shown in table 2. From table 3, with 5% probability, the daily rate of change of gasoil price could be as low as -0.624% and given that the rate of change is less than -0.624 the average rate of change value is -1.404%. Likewise, with 1% probability the daily rate of change of gasoil price could be as low as -1.3818%, and given this rate of change, the average rate of change value is -2.1870.

6. Conclusion

In this paper we have shown that quantile regression posses a more flexible appealing properties enough to capture the underlying complex dependence structure of a time series data. We have also illustrated how extreme value theory can be used to model tail related risk measures depending on data availability, frequency, desired time horizon and the level of complexity one is willing to introduce in the model. It can also be seen from the empirical study that this method can be used to determine the threshold by quantification.

7. Acknowledgements
We would like to thank The Kenya National Council for Science and Technology for their grant support towards this research. We would also like to thank The Kenya National Oil Corporation for enabling us access the data used for this study. Finally we wish to thank the reviewers and the editors for their comments and suggestion.

Table 2 ARMA-GARCH parameters, statistics of the standard residuals as well as POT parameters.

AR-GARCH parameters	
a_0	***
a_1	-0.0472 (0.0576)***
b_1	-0.6168 (0.0263)
$\varphi_0 \times 10^{-3}$	1.0576 (0.2733)
φ_1	0.1469 (0.0120)
φ_2	0.8758 (0.0090)
Standardized residuals descriptive statistics	
Mean	0.03256
Standard deviation	1.01179
Skewness	0.00122
Kurtosis	10.60099
Ljung P-value	0.97
POT parameters	
ξ	0.04635 (0.0393)
β	0.61109 (0.0493)
u	0.055

**** Not Significant parameter estimates. Figures in parenthesis are standard errors*

Table 3 Risk measures computed via Peak over threshold measure at 5%, 1% and 0.1% probability.

Probability	quantile	Shortfall
0.95	0.624	1.404
0.99	1.3818	2.1870
0.999	3.2574	4.7766

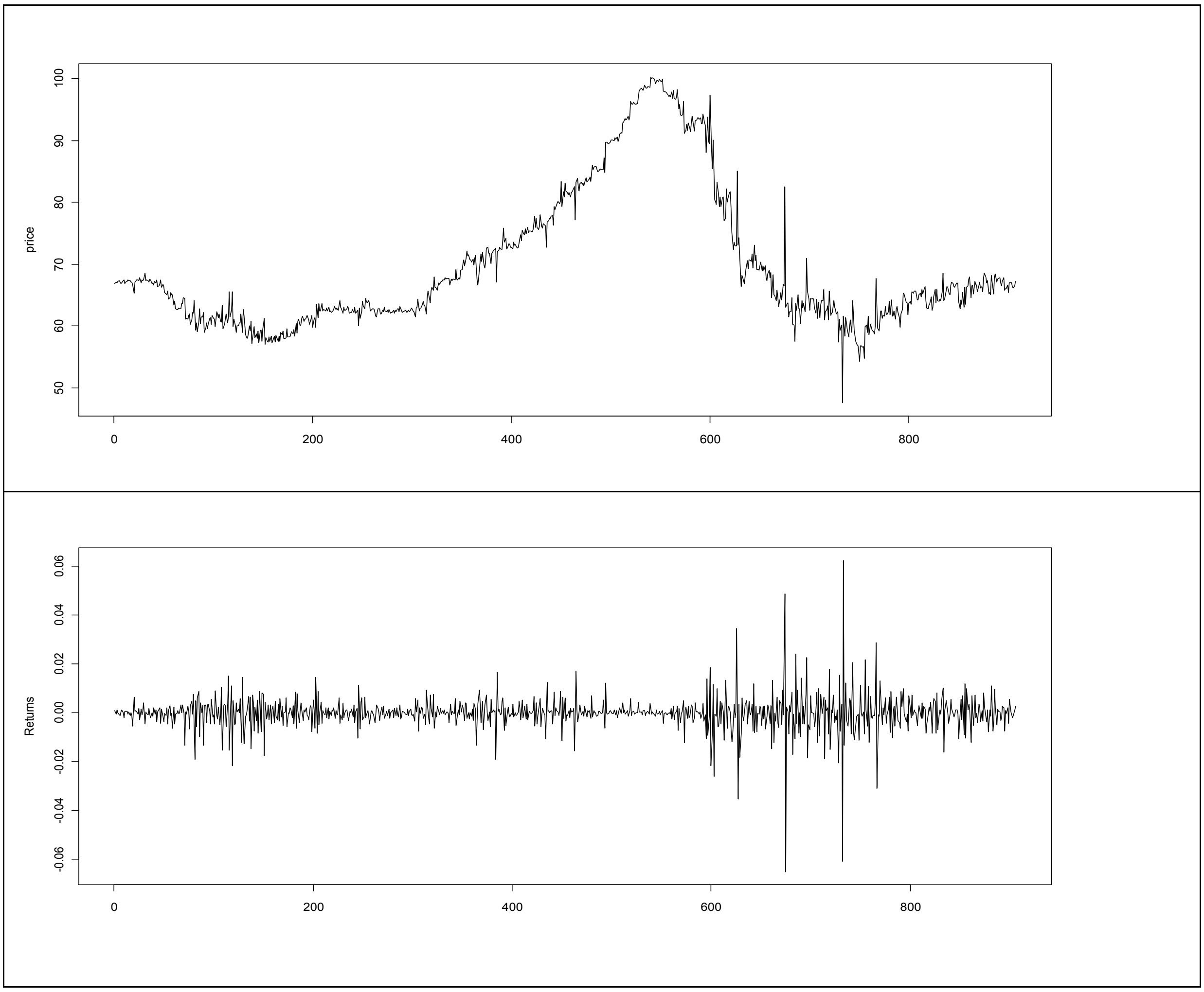

Figure1 Gasoil price changes (%) superimposed on the time series trend of gasoil daily prices.

8. References

[1] Byström H N E 2005 Extreme value theory and extremely large electricity price changes *Int. Rev. of Eco. Fin.* **14** (1) 41-55
[2] Sebitosi AB 2010 Is the South African electricity tariff model conducive to an energy efficient economy *En. for S. Dev. J.* **14** 315-319
[3] Bunn D W and Karakatsani N 2003 *Forecasting Electricity Prices* (London: London Business School)
[4] Danielsson J and deVries C G 1997 Tail index and quantile estimation with very high frequency data *J. Emp. Fin.* **4** 241-257
[5] Mbugua L, Mwita P and Mwalili S 2011 Application of nonparametric methods in studying energy consumption *Rwanda J.* **23** 7-20
[6] Koenker R W and Basset G W 1978 Regression quantiles *Econometrica* **46** 33-50
[7] Embrechts P, Klüppelberg C and Mikosch T 1997 *Modelling Extremal Events for Insurance and Finance* (Berlin: Springer)
[8] Pickands J 1975 Statistical inference using extreme order statistics *Ann. of stat.* **3** 119-131
[9] McNeil AJ and Frey R 2000 Estimation of tail-related risk measures for heteroscedastic financial time series an extreme approach *J. of Emp. Fin.* **7** 271-300