MACHAKOS UNIVERSITY

University Examinations 2018/2019

SCHOOL OF PURE AND APPLIED SCIENCES

DEPARTMENT OF MATHEMATICS, STATISTICS AND ACTUARIAL SCIENCE

THIRD YEAR SECOND SEMESTER EXAMINATION FOR

BACHELOR OF SCIENCE (MATHEMATICS AND COMPUTER SCIENCE)

BACHELOR OF SCIENCE (MATHEMATICS)

BACHELOR OF EDUCATION

BACHELOR OF ARTS

SMA 363 TESTS OF HYPOTHESIS

DATE: 25/4/2019 **TIME: 2:00 – 4:00 PM**

INSTRUCTIONS:

Answer Question ONE and Any Other Two Questions

QUESTION ONE (COMPULSORY) (30 MARKS)

a) Outline the general procedure used for hypothesis testing. (4 marks)

b) Differentiate between a simple hypothesis and composite hypothesis and give an example. (3 marks)

c) Differentiate between Type I and Type II errors as used in hypothesis testing. (4 marks)

d) Let X be a binomial random variable. We wish to test the hypothesis $H_0: p = 0.8$ against $H_a: p = 0.6$. Let $\alpha = 0.03$ be fixed. Find β for $n = 20$. (4 marks)

e) Use the Neyman-Pearson Lemma to obtain the best critical region for testing $\theta = \theta_0$ against $\theta = \theta_1 > \theta_0$, in the case of a normal population $N(\theta, \sigma^2)$, where σ^2 is known. (6 marks)

f) It is claimed that sports-car owners drive on the average 18,000 Km per year. A consumer firm believes that the average mileage is probably lower. To check, the consumer firm obtained information from 40 randomly selected sports car owners that resulted in a sample mean of 17,463 Km with a sample standard deviation of 1348Km. Formulate and test the hypothesis for the above claim. Take $\alpha = 0.01$. (5 marks)

g) Differentiate between a Most powerful test and the Uniformly most powerful test. (4 marks)

QUESTION TWO (20 MARKS)

a) A random variable X is believed to follow an $Exp(\lambda)$ distribution. In order to test the null hypothesis $\mu = 20$ against the alternative $\mu = 30$, where $\mu = \frac{1}{\lambda}$, a single value is observed from the distribution. If this value is less than 28, H_0 is accepted, otherwise H_0 is rejected. Find the probabilities of

i) a Type I error

ii) a Type II error (8 marks)

b) Let $X_1, \dots, X_n$ be a random sample from a Poisson distribution with mean λ. Derive the most powerful test for testing $H_0: \lambda = 3$ against $H_a: \lambda = 6$. (7 marks)

c) Let $X_1, \dots, X_n$ be a random sample from a $U(0, \theta)$ distribution. Find the most powerful α-level test for testing $H_0: \theta = \theta_0$ versus $H_a: \theta = \theta_1$, where $\theta_0 < \theta_1$. (5 marks)

QUESTION THREE (20 MARKS)

a) Let p be the probability that a coin will fall head in a single toss. In order to test the hypothesis $H_0: p = \frac{1}{2}$, the coin is tossed 6 times and the hypothesis H_0 is rejected if more than 4 heads are obtained. Find the probability of Type I error. If $H_a: p = \frac{3}{4}$, find the probability of Type II error. (6 marks)

b) Let X have a p.d.f. of the form

$$f(x, \theta) = \begin{cases} \frac{1}{\theta} \exp\left(-{x}/{\theta}\right), 0 < x < \infty, \theta > 0 \\ 0, elsewhere \end{cases}$$

To test $H_0: \theta = 2$ against $H_a: \theta = 1$ use a random sample X_1, X_2 of size 2 and define a critical region$C = \{(x_1, x_2): 9.5 \le x_1 + x_2\}$. Determine the power of the test.

(4 marks)

c) Let $X_1, \dots, X_n$ be a random sample from a $N(\mu, \sigma^2)$. Assume that σ^2 is known. Determine an appropriate likelihood ratio test to test $H_0: \mu = \mu_0$vs $H_a: \mu \neq \mu_0$at level α. (10 marks)

QUESTION FOUR (20 MARKS)

a) Define 'Likelihood ratio Test'. Under what circumstances would you recommend this test? (4 marks)

b) Let $x_1, \dots, x_n$ be a random sample from a normal distribution $N(\theta_1, \theta_2)$. Use the likelihood ratio test to obtain the best critical region of size α under $H_0: \theta_1 = 0$ against $H_1: \theta_1 \neq 0$. (8 marks)

c) The average blood pressure for a control group C of 10 patients was 77mmHg. The average blood pressure in a similar group T of 10 patients of a special diet was 75mmHg. Further, $\sum_{i=1}^{10} C_i^2 = 59{,}420$ and $\sum_{i=1}^{10} T_i^2 = 56{,}390$.

i) Write down the hypothesis you would test to check whether patients on the special diet have lower blood pressure. (2 marks)

ii) Compute the value of the test statistic and conclude appropriately at 5% significance level. (6 marks)

QUESTION FIVE (20 MARKS)

a) In order to increase the efficiency with which employees in a certain organisation can carry out a task, 5 employees are sent on a training course. The time in seconds to carry out the task both before and after the training course is given below for the 5 employees:

Employee	1	2	3	4	5
Before	42	51	37	43	45
After	38	37	32	40	48

Formulate the hypothesis and test whether the training course has had the desired effect. Take $\alpha = 0.05$. (8 marks)

b) Consider the data below on weekend sales of ice cream in Nairobi.

Temperature (X)	25	16	28	20	22	23	16	18
Sales(in 100 shillings) (Y)	125	79	140	103	111	115	80	91

i) Fit a simple linear regression model. (4 marks)

ii) Test the following hypothesis at 5% significance level

$$H_0: \beta_1 = 0 \; Vs \; H_1: \beta_1 \neq 0$$

and interpret the result obtained. (8 marks)