MACHAKOS UNIVERSITY

University Examinations 2018/2019

SCHOOL OF PURE AND APPLIED SCIENCES

DEPARTMENT OF MATHEMATICS, STATISTICS AND ACTUARIAL SCIENCE

SECOND YEAR FIRST SEMESTER EXAMINATION FOR

DOCTOR OF PHILOSOPHY IN FINANCE

BMS 919: ADVANCED STATISTICAL METHODS

DATE: **TIME:**

INSTRUCTIONS:

Attempt Question ONE and Any Other Two Questions

QUESTION ONE (COMPULSORY) (30 MARKS)

a) The following are values of a random sample from a population having a continuous distribution with median M.

1.5, 2.5, 2.7, 7.0, 4.5, 12.5, 11.0, 3.0

Test $H_o : M = 11$

Vs$H_1 : M < 11$

at 5% level of significance using the probability value of the

i) Sign test

ii) Wilcoxon's Signed-rank test (10 marks)

b) Given the following paired observations

X:	20	32	27	39	31	29	25
Y:	21	33	28	38	32	27	27

Test at 5% level of significance the hypothesis that the two samples have come from identical population using Wilcoxon's signed-rank test (10 marks)

c) Consider the AR MA(2, 2) process given by

$X_t - X_{t-1} - \propto X_{t-2} = e_{t-2} + e_{t-1} - 6\, e_t$ where $\propto$ is some constant and $\{e_t\}$ is a sequence of white noise? Show that the process is invertible. (5 marks)

d) Consider the process (X_t) given by $X_t = X_{t-1} + e_t$

where $\{e_t\}$ is the white noise process with mean zero and variance σ^2.

i) Find the mean and variance of the process

ii) Hence, or otherwise show that it is non-stationary (5 marks)

QUESTION TWO (20 MARKS)

a) Consider the time series given in Question one , part (d)

i) Show that the process

$Y_t = \Delta X_t$ is stationary.

ii) Determine the auto correlation function, and hence, the correlogram of (Y_t).

(6 marks)

b) Consider the process $Y_t = \varepsilon_t + 2\,\varepsilon_{t-1} + 0.8\, Y_{t-1}$ where $\{\varepsilon_t\}$ is sequence of independent random variables with mean 0 and variance σ^2. Determine the auto-correlation function ρ_k k = 0, 1, 2, 3,…… (8 marks)

c) Let $X_t = e_t \cos \lambda t + e_{t-1} \sin\lambda t$ where $\{e_t\}$ is white noise, λ is some constant. Show (X_t) is stationary. (2 marks)

QUESTION THREE (20 MARKS)

a) Consider the MA(1) process X_t given by $X_t = e_t + \theta e_{t-1}$ where $\{e_t\}$is the white noise process with zero mean and variance, σ^2. Show that the corresponding normalized spectral density function

$$f^*(\lambda) = \frac{1}{2\pi}\left\{1 + \frac{2\theta}{1+\theta^2} \, Cos\, \lambda\right\}$$

(8 marks)

b) Consider the AR (2) process

$$X_t = X_{t-1} - 0.5X_{t-2} + e_t$$

i) Determine whether the process is stationary

ii) Determine the auto correlation function for the process. (12 marks)

QUESTION FOUR (20 MARKS)

.Let $X_1, X_2, \ldots\ldots\ldots\ldots, X_m$ be a random sample from a continuous distribution function $F_x(x)$ and $Y_1, Y_2, \ldots\ldots\ldots\ldots\ldots\ldots, Y_n$ be a random sample from a continuous distribution function $F_y(x)$. To test $H_0: F_x(x) = F_y(x)$ for all x against a suitable alternative, a linear rank test statistic W_N is proposed.

Define the function

$$Z_i = \begin{cases} 1 & if\ i-th\ observation\ is\ an\ X \\ 0 & if\ i-th\ observation\ is\ a\ Y \end{cases}$$

a) Using the function Z_i or otherwise,

i) Define the Wilcoxon's sign rank test statistic W_N

ii) Explain how the statistic can be used for the median test (7 marks)

Define the function D_{ij} by

$$D_{ij} = \begin{cases} 1\ if\ Y_j < X_i \\ 0\ if\ Y_j > X_i \end{cases}$$

$i = 1, 2, \ldots\ldots\ldots\ldots\ldots\ldots, m : j = 1, 2, \ldots\ldots\ldots\ldots\ldots\ldots, n$

b) Using the function D_{ij} or otherwise,

(i) Define the Mann Whitney U statistic test statistic W_R , R=N+M

(ii) Explain how the statistic can be used for a 2-sample test. (7 marks)

c) Given that, W_R is a linear function of W_N show that

$$Var(W_R) = \frac{R}{24}(R+1)(2R+1)$$ (6 marks)

QUESTION FIVE (20 MARKS)

a) Consider the Wilcoxon's statistic W_N in Question Three. Let M denote the median of the distribution and consider the null hypothesis

$H_0: M = M_0$, where M_0 is known.

i) Show that when H_0 is true, for a random sample of size N,

$$E(W_N) = \frac{N(N+1)}{4}$$

iii) Hence, using the results given in Question Four or otherwise, given the approximate distribution of W_N when H_0 is true. (10 marks)

b) Suppose $(X_1, Y_1), (X_2, Y_2), \ldots, (X_n, Y_n)$ is a random sample from a continuous bivariate distribution with the distribution function F(x,y).

Let $D_i = Y_i - X_i$; $i = 1, 2, \ldots, n$

Assuming thet $D_1, D_2, \ldots, D_n$ is a random sample of difference from F(x,y), construct a $100(1 - \alpha)\%$ confidence interval for the unknown median of differences M_D based on the statistic T^+. (10 marks)