MACHAKOS UNIVERSITY

UNIVERSITY EXAMINATIONS 2018/2019

SCHOOL OF PURE AND APPLIED SCIENCES

DEPARTMENT OF MATHEMATICS, STATISTICS AND ACTUARIAL SCIENCE

THIRD YEAR SECOND SEMESTER EXAMINATIONS FOR

BACHELOR OF ECONOMICS AND STATISTICS

SMA 361: THEORY OF ESTIMATION

DATE: 18/4/2019 **TIME: 11:00 – 1:00 PM**

INSTRUCTION: *Answer Question **ONE** which is compulsory*

QUESTION ONE (COMPULSORY) (30 MARKS)

a) Explain the meaning of the following terms as applied in theory of estimation

(i) Population

(ii) An estimator (4 marks)

$b)$ $Let\ x_1, x_2, ..\, x_n$ be a random sample drawn from a population with mean μ

(unknown).Show that if $y = \sum_1^n a_i x_i$ is unbiased estimator for μ if $\sum_1^n a_i = 1$ (6 marks)

c) Let $x_1, x_2, ..\, x_n$ denote a random sample from a pdf is $P(X = x) = \frac{e^{-\mu}\mu^x}{x!}$, x=0,1,….

Use the factorization theorem to show that $T = \sum x_i$ is a sufficient estimator of μ

(4 marks)

d) $Let\ x_1, x_2, ..\, x_n$ be a random sample from a population with mean μ and variance δ^2.

Consider the following estimators forμ.

$$t_1 = {}^1\!/_2\,(x_1 + x_2)$$

$$t_2 = \frac{\frac{1}{2}x_1 + x_2 + \cdots + x_{n-1}}{2(n-1)}$$

$$t_3 = \bar{x}$$

i) Show that each of 3 estimators is unbiased. (5 marks)

ii) Determine the efficiency of t_3 relative to t_2 (5 marks)

e) Let $x_1, x_2, .. x_n$ be a random sample of size n from a normally distributed population with mean μ (unknown) and known variance δ^2.Determine the maximum likelihood estimator (MLE) of μ . (6 marks)

QUESTION TWO (20 MARKS)

a) Use the following data to fit the regression line $y = \propto + \beta x$

x	-2	-1	0	1	2
y	0	0	1	1	3

(8marks)

b) $Let\ x_1, x_2, .. x_n$ be a random sample of size n from a population whose pdf is given by

$f(x, \theta) = \theta e^{-\theta x}$, x> 0,

Determine the Cramer-Rao lower bound for θ. (12marks)

QUESTION THREE (20 MARKS)

a) Let $x_1, x_2, .. x_n$ be a random sample from $x \sim N(\mu_0, \delta^2)$ where μ, δ^2 are both unknown. Determine a joint sufficient statistic of $\mu, and\ \delta^2$ (8 marks)

b) A random sample of size n is drawn from a normal population with (0,δ^2). Determine the minimum variance unbiased estimator (MVUE) of δ^2 and its variance. (12 marks)

QUESTION FOUR (20 MARKS)

a) Let $x_1, x_2, .. x_n$ denote a random sample from a pdf

$$f(x) = \begin{bmatrix} (\theta + 1)x^{\theta}, & 0 < \text{x} < 1, \ \theta > 0 \\ 0 & elsewhere \end{bmatrix}$$

Obtain the moment estimator of θ (8 marks)

b) Given a random sample of size n from a population whose pdf is

$$f(x) = \frac{1}{2\sqrt{2\pi\theta}} e^{\frac{-1}{4\theta}(x-5)^2}$$

Obtain the maximum likelihood estimator (MLE) of θ and show that it's unbiased.

(12 marks)

QUESTION FIVE (20 MARKS)

a) A population has a known mean μ and a standard deviation δ of 2.45. If a sample of 900 has a mean of 3.15cm and the population is normal obtain a 99% confidence intervals for the true mean. (8 marks)

b) If $x_1, x_2, .. x_n$ is a random sample of size n from a population whose pdf is given by

$$f(x, \lambda) = \frac{e^{-\lambda}\lambda^x}{x!}, x = 0, 1, 2, ...$$

Determine the uniformly minimum variance unbiased estimator (UMVUE) of $e^{-\lambda}$ (12 marks)