MACHAKOS UNIVERSITY

UNIVERSITY EXAMINATIONS 2018/2019

SCHOOL OF PURE AND APPLIED SCIENCES

DEPARTMENT OF MATHEMATICS, STATISTICS AND ACTUARIAL SCIENCE

SECOND YEAR FIRST SEMESTER EXAMINATION FOR

BACHELOR OF SCIENCE (MATHEMATICS AND COMPUTER SCIENCE)

SMA 405: GROUP THEORY II

DATE: 10/5/2019 **TIME: 2:00 – 4:00 PM**

INSTRUCTION: *Answer Question **ONE** which is compulsory*

QUESTION ONE (COMPULSORY) (30 MARKS)

a) Differentiate between a centre of a group and centralizer (4 marks)

b) Prove that p –group G is nilpotent (4 marks)

c) Show that the alternating group A_4 has no subgroup of order 6 (5 marks)

d) Show that if H is a subset of a group G and $g \in G$, then $|g^{-1}Hg| = |H|$, where $g^{-1}Hg = \{g^{-1}hg | h \in H\}$ (5 marks)

e) State without proving the second Sylow theorem (2 marks)

f) Prove if G is a group of order p^r,$r \geq 1$, then G has a normal subgroup of order p^{r-1} (5 marks)

g) Find all Sylow p-subgroups of A_4 for *p=2* and *3*. (5 marks)

QUESTION TWO (20 MARKS)

a) Prove that if G is a group with subgroups H and K such that $H \cap K = \{1\}$, the elements of H commute with those of K,and HK=G. Then $G \cong H \times K$ (8 marks)

b) Prove G is a finite abelian group , G is solvable (5 marks)

c) Prove that $G = H \times K$ is the direct product of the groups H and K, then the sets

$\mathcal{H} = \{(h, 1)|h \in H, 1 \ \ the\ identity\ of\ H\ \}$

$\mathcal{K} = \{(1, k)|k \in K, 1 \ \ the\ identity\ of\ K\ \}$ are subgroups of G. (5 marks)

d) State the Jordan-Holder theorem (2 marks)

QUESTION THREE (20 MARKS)

a) Prove that if $p \neq \emptyset$ is a subset of G and $\mathcal{A} = \{g^{-1}pg|g \in G\}$. Then $|\mathcal{A}| = \sum_{R \in \mathcal{R}}[H: N_H\ (R)] = [G: N_G\ (P)]$ (10 marks)

b) Prove that a finite group G is a p-group if and only if every element of G has order a power of p (5 marks)

c) Show that S_4 is solvable (5 marks)

QUESTION FOUR (20 MARKS)

a) State and prove first Sylow theorem (10 marks)

b) Prove if G is a finite group, P a Sylow of G, and H is a subgroup of G of order a power of P, then $N_H(P) = H \cap P$ (5 marks)

c) Show that the symmetric group S_n is solvable for n=1,2,3. (5 marks)

QUESTION FIVE (20 MARKS)

a) Prove that if G is a finite group with subgroup H and non-empty subset A, the number of distinct subsets of G with distinct H-conjugates of A is the index of $N_H(A)$ in H (8 marks)

b) State without proving the third Sylow theorem (2 marks)

c) Prove that A_4 is not nilpotent (2 marks)

d) Prove that if two groups have the same composition factors, are they isomorphic? (3 marks)

e) Prove that any subgroup of a solvable group is solvable (5 marks)