MACHAKOS UNIVERSITY

UNIVERSITY EXAMINATIONS 2018/2019

SCHOOL OF PURE AND APPLIED SCIENCES

DEPARTMENT OF MATHEMATICS, STATISTICS AND ACTUARIAL SCIENCE

DECEMBER SESSION EXAMINATION FOR

BACHELOR OF EDUCATION

SMA 203: LINEAR ALGEBRA II

DATE: 25/4/2019 **TIME:**

INSTRUCTION: *Answer Question **ONE** which is compulsory and any other **TWO** Questions*

QUESTION ONE (COMPULSORY (30 MARKS)

a) Find a basis for the subspace W of $\mathbf{R}^4$ generated by the set $\{(1,2,3,1),(4,1,3,2),(6,5,9,4)\}$

(4 marks)

b) Let $T:\mathbf{R}^2 \to \mathbf{R}^2$ defined by $T(x,y)=(2y,2x-y)$ relative to the standard basis $e_1=(1,0), e_2=(0,1)$ of $\mathbf{R}^2$ (4 marks)

c) Compute $\det A$ where

$$A=\begin{bmatrix} 4 & 0 & 0 & 0 \\ 7 & -1 & 0 & 0 \\ 2 & 6 & 3 & 0 \\ 5 & -8 & 4 & 3 \end{bmatrix}$$ (5 marks)

e) Let $A=\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$. Find all the eigen values of A and a basis for each eigen space. (5 marks)

f) Consider the following basis of $\mathbf{R}^3$

$\{e_1 = (1,0,0), e_2 = (0,1,0), e_3 = (0,0,1)\}$

$f_1 = \{(1,1,1), f_2 = (1,1,0), f_3 = (1,0,0)\}$

i) For $v = (a,b,c) \in \mathbf{R}^3$, find $[v]_e$ and $[v]_f$ (5 marks)

ii) Show that $[T]_f = P^{-1}[T]_e P$ where T is defined by $T(x,y,z) = (2y+z, x-4y, 3x)$

(7 marks)

QUESTION TWO (20 MARKS)

a) Show that the mapping $T : \mathbf{R}^3 \to \mathbf{R}^2$ define by $T(x,y,z) = (2x-y, y-5z)$ is a linear transformation. (4 marks)

b) Let $T(x,y) = (x+4y, -2x+3y)$ and let $\{e_1 = (1,0), e_2 = (0,1)\}$ and $\{f_1 = (1,1), f_2 = (2,-1)\}$ be two basis $\mathbf{R}^2$

i) Find the transition matrix P from $\{e_1\}$ to $\{f_1\}$ and the transition matrix $Q = P^{-1}$ from $\{f_1\}$ to $\{e_1\}$, (5 marks)

ii) Show that $P^{-1}[T]_e P = [T]_f$ is a diagonal matrix of T i.e. T is diagonalizable. (5 marks)

c.) Let $A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$, $f(t) = 2t^2 - 3t + 7$ and $g(t) = t^2 - 5t - 2$ Find $f(A)$ and $g(A)$. (6 marks)

QUESTION THREE (20 MARKS)

a) Show that $T : \mathbf{R}^2 \to \mathbf{R}^2$, defined by $T(x,y) = (x-2y, 2x+3y)$ is one-one and onto.

(5 marks)

b) Find the characteristics equation for the matrix $T = \begin{bmatrix} 3 & 0 & 1 \\ 0 & 2 & 0 \\ 5 & 0 & -1 \end{bmatrix}$ (5 marks)

c) Find the eigen values and eigen vectors associated with the matrix $\begin{bmatrix} 1 & 2 \\ 3 & 2 \end{bmatrix}$. (10 marks)

QUESTION FOUR (20 MARKS)

Let A be the matrix $\begin{bmatrix} 1 & 0 & 0 & 3 & 2 & 2 \\ 4 & 1 & 3 & 3 & 5 & 3 \\ 6 & 7 & 12 & 0 & 0 & 4 \\ 5 & 1 & 3 & 6 & 7 & 5 \end{bmatrix}$

i) Use elementary row operations to reduce the matrix A to form $\begin{bmatrix} I & B \\ 0 & 0 \end{bmatrix}$ where I is 3 by 3 identify matrix and 0 denote the 1 by 3 zero matrix. (4 marks)

ii) Write down a basis for the row space of A (4 marks)

iii) Verify that the matrix T satisfies the characteristics equation and hence find T^{-1} (the inverse of T). (6 marks)

iv) Find a matrix p that diagonalizes the matrix T and determine the product $P^{-1}TP$. (6 marks)

QUESTION FIVE (20 MARKS)

a) Write down a basis for the image of $\mathbf{R}^3$ under T . (2 marks)

b) Consider the polynomial $f(x) = 2 + 3x - 4x^2 + 2x^3$. Find the co-ordinates of $f(x)$ with respect to the basis $\{1, (1+x), (1+x+x^2), (1+x+x^2+x^3)\}$ (5 marks)

c) Compute $\det A$ by Gauss elimination method, where

$$A = \begin{bmatrix} 2 & -8 & 6 & 8 \\ 3 & -9 & 5 & 10 \\ -3 & 0 & 1 & -2 \\ 1 & -4 & 0 & 6 \end{bmatrix}$$ (6 marks)

d) A linear transformation $T : \mathbf{R}^3 \to \mathbf{R}^3$, defined by $T(x_1, x_2, x_3) = (x_3, x_1 + x_2, x_1 + x_2 + x_3)$. Find the rank, nullity, Kernel and basis of $KerT$ and $\operatorname{Im} T$ (7 marks)