MACHAKOS UNIVERSITY

University Examinations 2018/2019

SCHOOL OF PURE AND APPLIED SCIENCES

DEPARTMENT OF MATHEMATICS, STATISTICS AND ACTUARIAL SCIENCE

FIRST YEAR SPECIAL/ SUPPLEMENTARY EXAMINATION FOR

BACHELOR OF SCIENCE IN STATISTICS AND PROGRAMMING

SST 103: LINEAR ALGEBRA I

DATE:25/9/2019 **TIME: 11:00 – 1:00 PM**

INSTRUCTIONS:

Answer QUESTION <u>ONE</u> and any other <u>TWO</u> QUESTIONS

QUESTION ONE 30 Marks (Compulsory)

a) Calculate the cross product of the vectors $\vec{U}$ = (1, 2, 3) and $\vec{v}$ = (−1, 1, 2). (3 marks)

b) Find the angle between $u = 2i$ +2j+2k and $v = i + j + k$ (3 marks)

c) Find the inverse of the following matrix

$$\begin{bmatrix} 0 & 2 & 4 \\ 2 & 4 & 2 \\ 3 & 3 & 1 \end{bmatrix}$$ (4 marks)

d) Solve the linear system

$$x + 2y + 3z = 4$$
$$3x + 8y + 7z = 20$$
$$2x + 7y + 9z = 23$$

(5 marks)

e) Find the equation of the plane through the point (-1, 2, 3) and perpendicular to the plane $2x - 3y + 4z = 1$ and $3x - 5y + 2z = 3$ (5 marks)

f) Determine c so that $a = ci + 5j - 3k$ and $b = 3i + 4j - 2k$ are orthogonal (5 marks)

g) Find the parametric and symmetric equation of the line passing through the point (2, -3, 4) and parallel to the vector (3, 5, -6) (5 marks)

Question two 20 marks

a) Show that W= $\{(x, y) \,/\, x = 2y\}$ is a subspace for R^2. (4 marks)

b) Prove that the diagonals of a rhombus are perpendicular. (5 marks)

c) Find the parametric and the symmetric equations of the line passing through the point $(2, 3, -4)$ and parallel to the vector $(3,5,-6)$ (5 marks)

d) Define a vector space (6 marks)

QUESTION THREE 20 MARKS

a) Find the determinant of the following matrix

$$\begin{bmatrix} 2 & 5 & 0 \\ 1 & 4 & 8 \\ 4 & 7 & 1 \end{bmatrix}$$ (3 marks)

b) Find the ranks of the following matrix $\begin{bmatrix} 1 & 3 & 1 \\ 2 & 3 & 2 \\ 1 & 3 & 0 \end{bmatrix}$ (5 marks)

c) Find the minors and cofactors of the following matrix $\begin{bmatrix} 2 & 1 & 2 \\ 2 & 3 & 1 \\ 1 & 3 & 1 \end{bmatrix}$ (6 marks)

d) Transpose the following matrix $\begin{bmatrix} 2 & 1 & 3 \\ 2 & 3 & 7 \\ 1 & 4 & 3 \end{bmatrix}$ (2 marks)

e) Reduce the following matrix into echelon form $\begin{bmatrix} 1 & 2 & 4 \\ 2 & 1 & 4 \\ 1 & 6 & 1 \end{bmatrix}$ (5 marks)

QUESTION FOUR 20 MARKS

a) Solve the following system of equation by elimination method

$$x_1 + x_2 + x_3 = 5$$

$$x_1 + 2x_2 + 3x_3 = 10$$

$$2x_1 + x_2 + x_3 = 6$$ (4 marks)

b) Solve the simultaneous equation using the gauss-Elimination method

$$2x_1 - 4x_2 + 6x_3 = 20$$

$$6x_1 - 12x_2 + 2x_3 = 44$$

$$-4x_1 + 10x_2 - 4x_3 = -36$$

(5 marks)

c) Solve by Crammer's Rule

$$x - 2y + 3z = 10$$

$$3x - 6y + z = 22$$

$$-2x + 5y - 2z = -18$$

(5 marks)

d) Determine the value of 'a' so that the following systems in unknown *x*, *y* and *z* has

i) No solution

ii) More than one solution

iii) Unique solution

$$x - 3z = -3$$

$$2x + ay - z = -2$$

$$x + 2y + az = 1$$

(6 marks)

QUESTION FIVE 20 MARKS

a) Find the equation of the plane passing through the point $(3, -1, 7)$ and perpendicular to the vector $n = (4, 2, -5)$. (5 marks)

b) Find the distance D between the point (1, -4, -3) and the plane $2x - 3y + 6z = -1$ (5 marks)

c) Find the parametric equations for the line of intersection of the plane $3x + 2y - 4z - 6 = 0$ and $x - 3y - 2z - 4 = 0$ (5 marks)

d) Show that the vectors u=(1,-1,0) v=(1,3,-1) and w=(5,3,-2) are linearly dependent

(5 marks)