MACHAKOS UNIVERSITY

University Examinations 2018/2019

SCHOOL OF PURE AND APPLIED SCIENCES

DEPARTMENT OF MATHEMATICS, STATISTICS AND ACTUARIAL SCIENCE

THIRD YEAR SPECIAL/ SUPPLEMENTARY EXAMINATION FOR
BACHELOR OF SCIENCE (ELECTRICAL AND ELECTRONICS)
BACHELOR OF SCIENCE (MECHANICAL ENGINEERING)
BACHELOR OF SCIENCE (CIVIL ENGINEERING)
ECU 201: ENGINEERING MATHEMATICS VI
ECU 203: ENGINEERING MATHEMATICS VIII

DATE: 24/7/2019 **TIME: 8:30 – 10:30 AM**

INSTRUCTIONS

Attempt question ***one (compulsory)*** and any other ***two questions***.

QUESTION ONE (COMPULSORY)(30 MARKS)

a) Calculate $P_4(x)$ using the Rodrigues formula for the Legendre functions. (5 marks)

b) Determine the polynomial solution to the Legendre equation

$$(1-x^2)y'' - 2xy' + 12y = 0$$ (5 marks)

c) Evaluate $\int J_5(x)dx$ (5 marks)

d) Using the Bessel function identities, express $[J_2(x)]'$ in terms of $J_0(x)$ *and* $J_1(x)$.

(5 marks)

e) Determine the Laplace transform for;

i.) $f(t) = e^{at}$ (5 marks)

ii.) $f(x) = \cosh 2t$ (5 marks)

QUESTION TWO (20 MARKS)

a) Use Laplace transforms method to solve the second order differential equation $\frac{d^2x}{dt^2} - 3\frac{dx}{dt} + 2x = 2e^{3t}$ given that at $t = 0$; $x = 5$ *and* $\frac{dx}{dt} = 7$ (12 marks)

b) Determine the inverse of $\frac{5s+1}{(s-4)(s+3)}$ (8 marks)

QUESTION THREE (20 MARKS)

a) Use the Bessel's function identities to express $J_3(x)$ in terms of $J_0(x)$ *and* $J_1(x)$ (5 marks)

b) Evaluate $\int x^3 J_0 dx$ (5 marks)

c) Determine $J_{\pm\frac{3}{2}}$ in terms of the elementary functions $\sin x$ *and* $\cos x$ (4 marks)

d) Determine the general solution to Bessel's equation of order zero. (6 marks)

QUESTION FOUR (20 MARKS)

a) Determine the Fourier's series expansion for;

$$f(x) = \begin{cases} 0 & ; -\pi \le x < \frac{-\pi}{2} \\ \cos x & ; \frac{-\pi}{2} \le x < \frac{\pi}{2} \\ 0 & ; \frac{\pi}{2} \le x < \pi \end{cases}$$

(4 marks)

b) Determine a Fourier cosine series for $f(x) = e^x$ on $(0, \pi)$ (4 marks)

c) i.) Define a Dirac delta function (1 mark)

ii.) State the convolution theorem of Fourier transforms. (3 marks)

iii.) Given the functions $f(t)$ and $g(t)$ and their convolution is;

$$f(t) * g(t) = \int_{-\infty}^{\infty} f(x)g(t-x)dx = h(t)$$

Where $*$ denotes the convolution operation.

If $f(t) = u(t)$ and $g(t) = \begin{cases} \sec^2 t & |t| < \frac{\pi}{4} \\ 0 & otherwise \end{cases}$

Where $u(t)$ is the Heaviside function.

Show that $u(t)=\dfrac{1+\tan^2 t}{1+\tan t}$ (4 marks)

d.) Evaluate the inverse transform of $f(\omega)=\dfrac{1}{2\pi(a+j\omega)^2}$ where $a>0$

(5 marks)

QUESTION FIVE (20 MARKS)

a) Consider the expression $f(t)=L^{-1}\left\{\dfrac{2}{s}+\dfrac{3e^{-s}}{s^2}-\dfrac{3e^{-3s}}{s^2}\right\}$

i.) Determine $f(t)$ (6 marks)

ii.) Sketch the graph of $f(t)$ (7 marks)

b) When $p=n$ an integer $\alpha_1=\alpha_2=n$; $J_n(x)$ *and* $J_{-n}(x)$ and are linearly dependent. Show that $J_{-n}(x)=(-1)^n J_n(x)$ (7 marks)