MACHAKOS UNIVERSITY

University Examinations 2018/2019

SCHOOL OF PURE AND APPLIED SCIENCES

DEPARTMENT OF MATHEMATICS, STATISTICS AND ACTUARIAL SCIENCE

SECOND YEAR SPECIAL/ SUPPLEMENTARY EXAMINATION FOR

BACHELOR OF SCIENCE (ELECTRICAL AND ELECTRONICS)

BACHELOR OF SCIENCE (MECHANICAL ENGINEERING)

BACHELOR OF SCIENCE (CIVIL ENGINEERING)

ECU 202: ENGINEERING MATHEMATICS VII

DATE: 24/9/2019 **TIME: 8:30 – 10:30 AM**

INSTRUCTION TO CANDIDATES: ANSWER QUESTION ONE AND ANY TWO OTHER QUESTIONS

QUESTION ONE COMPULSORY (30 MARKS)

a) Form the differential equation associated with $y^2 = 4ax$ (3 marks)

b) Determine the differential equation of the family $y = Ae^{2x} + Be^{-2x}$ (5 marks)

c) Determine the general solution of $x\left(\frac{dy}{dx}\right)^2 + (y-x)\frac{dy}{dx} - y = 0$ (5 marks)

d) Apply the method of separation of variables to solve $\frac{dy}{dx} + xy = xy^3$ (5 marks)

e) Use the method of undetermined coefficient to solve

$$\frac{d^2y}{dx^2} - \frac{dy}{dx} - 2y = 4x^2$$ (5 marks)

f) A chain coiled up near the edge of a smooth table begins to fall over the edge. When a length x of the chain has fallen, the equation of motion is given by $\frac{d}{dt}(mxv) = mxg$, when m is the mass of the chain per unit length, v is the speed, g is the acceleration due to gravity and t is the time. Show that the speed is given by

$$v = \sqrt{\left(\frac{2}{3}\right)gx}$$ (7 marks)

QUESTION TWO (20 MARKS)

a) Show that

$Ax^2 + By^2 = 1$ is the solution of

$$x\left[y\frac{d^2y}{dx^2} + \left(\frac{dy}{dx}\right)^2\right] - y\frac{dy}{dx} = 0$$ (3 marks)

b) Use the method of undetermined coefficient to solve

$$\frac{d^2y}{dx^2} + 2\frac{dy}{dx} + 4y = 111e^{2x}\cos 3x$$ (7 marks)

c) Determine the orthogonal trajectories of the family

$$\frac{x^2}{a^2} + \frac{y^2}{a^2+\lambda} = 1$$ (10 marks)

QUESTION THREE (20 MARKS)

a) Solve $D^2 + 2D + 1 = x\cos x$ (6 marks)

b) Obtain the equation of the curve satisfying this equation and passing through the origin $\frac{dy}{dx} + \frac{2x}{1+x^2}y = \frac{4x^2}{1+x^2}$ (6 marks)

c) A cyclist, moving on a level road at $4m/s$ stops pedaling and free wheels to rest. The retardation of the cycle has two components a constant $0.08m/s^2$ due to friction in the working parts and resistance of $0.02v^2/s^2$ where v is the speed in meters per second. What distance traversed by the cyclist before it comes to rest? (8 marks)

QUESTION FOUR (20 MARKS)

a) Determine the general solution to $(D^4 - 81)y = 0$ (4 marks)

b) Determine the general solution of

$y = x + 2\tan^{-1} p$ (6 marks)

c) Reduce the given differential equation

$x^2\left(\frac{dy}{dx}\right)^2 + y(2x+y)\frac{dy}{dx} + y^2 = 0$ to Clairut's form by using the substitution $y = u$, $xy = v$, and determine its singular solution. (10 marks)

QUESTION FIVE (20 MARKS)

a) Solve

$$(2x+3y-5)\frac{dy}{dx}+(3x+2y-5)=0$$ (10 marks)

b) If the population of a country doubles in 50 years. In how many years will it triple under the assumption that the rate of increase is proportional to the number of inhabitants? (10 marks)