



# MACHAKOS UNIVERSITY COLLEGE

(A Constituent College of Kenyatta University)

UNIVERSITY EXAMINATIONS 2015/2016

THIRD YEAR EXAMINATION FOR THE DEGREE OF BACHELOR OF EDUCATION AND  
BACHELOR OF SCIENCE IN MATHEMATICS, COMPUTER SCIENCE & MATHEMATICS  
AND STATISTICS & PROGRAMMING.

**SMA 330-NUMERICAL ANALYSIS**

**DATE:** \_\_\_\_\_

**TIME: 2 HOURS**

## INSTRUCTION TO CANDIDATES

ANSWER QUESTION ONE AND ANY TWO OTHER QUESTIONS

### QUESTION ONE COMPULSORY (30 MARKS)

- a.) Convert the decimal number 81.5 to its binary form. (2 marks)
- b.) Show that the operators  $\mu$  and  $E$  commute. (3 marks)
- c.) Solve  $x^3 - 9x + 1 = 0$  for the root between  $x = 2$  and  $x = 4$ , by bisection method. (5 marks)
- d.) prove that the forward difference of the quotient of two functions is given by
- $$\Delta \left( \frac{f(x)}{g(x)} \right) = \frac{g(x)\Delta f(x) - f(x)\Delta g(x)}{g(x+h)g(x)} \quad (5 \text{ marks})$$
- e.) Set up a Newton iteration for computing the square root of a given positive number. Using the same find the square root of 2 exact to six decimal places. (7 marks)
- f.) find a root of the equation  $x^3 + x^2 - 1 = 0$  on the interval  $[0,1]$  with an accuracy of  $10^{-4}$  (8 marks)

**QUESTION TWO (20 MARKS)**

a.) If  $E, \mu$  and  $\delta$  denote shift, average and central difference operators, in analysis of data with equal spacing  $h$ , prove the following

$$E^{1/2} = \mu + \frac{\delta}{2} \quad (2 \text{ marks})$$

b.) Solve the equation  $x^3 = \text{Sin}x$ . Considering various  $\phi(x)$ , discuss the convergence of the solution. (5 marks)

c.) for the following table of values, estimate  $f(7.5)$ , using Newton's backward difference interpolation formula (5 marks)

e.) using the values given in the following table, find  $\text{Cos}0.28$  by linear interpolation and by quadratic interpolation and compare the results with the value 0.96106 (exact to 5D)

(8 marks)

**QUESTION THREE (20 MARKS)**

a.) Using Aitken's scheme and the following values evaluate  $\log_{10} 301$

$x$	300	304	305	307
$\log_{10} x$	2.4771	2.4829	2.4843	2.4871

(4 marks)

b.) find the positive solution of the transcendental equation

$$2\text{Sin}x = x \quad (5 \text{ marks})$$

c.) Find the cubic polynomial which takes the following values;  $f(1)=24, f(3)=120, f(5)=336$ , and  $f(7)=720$ . hence or otherwise, obtain the value of  $f(8)$ .

(5 marks)

d.) Evaluate  $\int_0^1 e^{-x^2} dx$  by means of Trapezoidal rule with  $n = 10$  (6 marks)

**QUESTION FOUR (20 MARKS)**

a.) Construct the forward difference table, where  $f(x) = \frac{1}{x}, x = 1(0.2)2,4D$  (4 marks)

b.) Find  $\int_0^{10} \frac{1}{1+x^2} dx$  using Simpson's one third rule. (7 marks)

c.) find the interpolating polynomial by Newton's divided formula for the following table and then calculate  $f(2.1)$ .

$x$	0	1	2	4
$f(x)$	1	1	2	5

(9 marks)

**QUESTION FIVE (20 MARKS)**

a.) Derive the Newton-Raphson iterative formula

$$x_n = x_{n-1} - \frac{f(x_{n-1})}{f'(x_{n-1})}, n \geq 1$$

For the solution of the non-linear equation  $f(x) = 0$ . (10 marks)

b.) Use the method in a.) to find the root of the equation  $\tan x = x$  near  $x = 4.5$  correct to four decimal places. (10 marks)