MACHAKOS UNIVERSITY

University Examinations 2018/2019

SCHOOL OF PURE AND APPLIED SCIENCES

DEPARTMENT OF MATHEMATICS, STATISTICS AND ACTUARIAL SCIENCE

THIRD YEAR SUPPLEMENTARY EXAMINATION FOR
BACHELOR OF SCIENCE(MATHEMATICS)
BACHELOR OF SCIENCE (STATISTICS AND PROGRAMMING)
SMA 363 TESTS OF HYPOTHESIS

DATE:26/9/2019 **TIME: 11:00 – 1:00 PM**

INSTRUCTIONS TO CANDIDATES

Answer Question ONE and ANY OTHER TWO

QUESTION ONE (30 Marks)

a) Explain briefly what is meant by hypothesis testing as used in statistics. (2 Marks)
b) Differentiate between a simple hypothesis and composite hypothesis and give an example. (3 Marks)
c) Differentiate between Type I and Type II errors as used in hypothesis testing. (4 Marks)
d) Let X be a binomial random variable. We wish to test the hypothesis $H_0: p = 0.8$ against $H_a: p = 0.6$. Let $\alpha = 0.03$ be fixed. Find β for $n = 10$. (4 Marks)
e) Let p be the probability that a coin will fall head in a single toss. In order to test $H_0: p = \frac{1}{2}$ against $H_1: p = \frac{3}{4}$, the coin is tossed five times and H_0 is rejected if more than three heads are obtained. Find the probability of Type I error and the power of the test. (6 Marks)
f) The average IQ of a sample of 50 University students was found to be 105. Carry out a statistical test to determine whether the average IQ of University students is greater than 100, assuming that IQ's are normally distributed. It is known from previous studies that the standard deviation of IQ's among students is approximately 20. (5 Marks)
g) Use the Neyman-Pearson Lemma to obtain the best critical region for testing $\theta = \theta_0$ against $= \theta_1 < \theta_0$, in the case of a normal population $N(\theta , \sigma^2)$, where σ^2 is known.

(6 Marks)

QUESTION TWO (20 Marks)

a) Outline the general procedure used for hypothesis testing. (4 Marks)

b) Differentiate between a Most powerful test and the Uniformly most powerful test. (4 Marks)

c) Let $X_1, \ldots, X_n$ denote an independent random sample from a population with a Poisson distribution with mean λ. Derive the most powerful test for testing $H_0: \lambda = 2$ against $H_a: \lambda = \frac{1}{2}$. (7 Marks)

d) In a certain stretch of a highway, where the speed limit is 70Km/h, it is thought that people travel on the average of at least 75Km/h. To check this claim, the following radar measurements of the speeds (in Km/h) is obtained for 10 vehicles travelling on this stretch of the intercounty highway.

66 74 79 80 69 77 78 65 79 81

Do the data provide sufficient evidence to indicate that the mean speed at which people travel on this stretch of highway is at most 75Km/h? Test the appropriate hypothesis at $\alpha = 0.01$. (5 Marks)

QUESTION THREE (20 MARKS)

a) Given the frequency function $f(x, \theta) = \begin{cases} \frac{1}{\theta}, 0 \leq x \leq \theta \\ 0, \; elsewhere \end{cases}$ and you are testing $H_0: \theta = 1$ versus $H_1: \theta = 2$ by means of a single observed value of x. What would be the sizes of Type I and Type II errors, if you choose the interval

i. $0.5 \leq x$ as the critical region. (3 Marks)

ii. $1 \leq x \leq 1.5$ as the critical region. (3 Marks)

b) In a one year investigation of claim frequencies for a particular category of motorists, the total number of claims made under 5,000 policies was 800. Assuming that the number of claims made by individual motorists has a Poisson (λ) distribution, test at the 1% level whether the unknown average claim frequency λ is less than 0.175. (4 Marks)

c) Suppose we have a random sample of size 25 from a normal population with an unknown mean μ and a standard deviation of 4. We wish to test the hypothesis $H_0: \mu = 10$ versus $H_a: \mu > 10$. Let the rejection region be defined by; reject H_0 if the sample mean $\bar{x} > 11.2$.

i. Find α (3 Marks)

ii. Find β for $H_a: \mu = 11$ (3 Marks)

iii. What should the sample size be if $\alpha = 0.01$ and $\beta = 0.8$ (4 Marks)

QUESTION FOUR (20 MARKS)

a) Define 'Likelihood ratio Test'. Under what circumstances would you recommend this test? (4 Marks)

b) Let $x_1, \ldots, x_n$ be a random sample from a normal distribution $N(\mu, \sigma^2)$. Assume that σ^2 is known. Find an appropriate likelihood ratio test to test at level α , $H_0: \mu = \mu_0$ against $H_1: \mu \neq \mu_0$. (8 Marks)

c) A car manufacturer runs tests to investigate the fuel consumption of cars using a newly developed fuel additive. Sixteen cars of the same make and age are used, eight with the new additive and eight as controls. The results, in miles per gallon over a test track under regulated conditions are as follows:

Control	27.0	32.2	30.4	28.0	26.5	25.5	29.6	27.2
Additive	31.4	29.9	33.2	34.4	32.0	28.7	26.1	30.3

If μ_c is the mean number of miles per gallon achieved by cars in the control group, and μ_A is the mean number of miles per gallon achieved by cars in the group with fuel additive, test at $\alpha = 0.05$ significance level, $H_0: \mu_A - \mu_C = 0$ versus $H_A: \mu_A - \mu_C > 0$ (8 Marks)

QUESTION FIVE (20 Marks)

a) It is desired to investigate the level of premium charged by two companies for contents policies for houses in a certain area. Random samples of 10 houses insured by Company A are compared with 10 similar houses insured by Company B. Premiums charged in each case are as follows:

Company A	117	154	166	189	190	202	233	263	289	331
Company B	142	160	166	188	221	241	276	279	284	302

For these data: $\sum A = 2{,}134$, $\sum A^2 = 494{,}126$, $\sum B = 2{,}259$, $\sum B^2 = 541{,}463$. Test at 5% significance level, whether the level of premiums charged by Company B are higher than those charged by Company A. (8 Marks)

b) Use the method of least squares to fit a straight line to the data points below:

x	-1	0	2	-2	5	6	8	11	12	-3
y	-5	-4	2	-7	6	9	13	21	20	-9

i. Estimate β_0 and β_1. (4 Marks)

ii. Test the following hypothesis at 5% significance level

$$H_0: \beta_1 = 2 \; Vs \; H_1: \beta_1 \neq 2$$

and interpret the result obtained. (8 Marks)