MACHAKOS UNIVERSITY

University Examinations 2018/2019

SCHOOL OF PURE AND APPLIED SCIENCES

DEPARTMENT OF MATHEMATICS, STATISTICS AND ACTUARIAL SCIENCE

FOURTH YEAR SECOND SEMESTER EXAMINATION FOR

BACHELOR OF EDUCATION SCIENCE

BACHELOR OF EDUCATION ARTS

BACHELOR OF SCIENCE (MATHEMATICS)

BACHELOR OF SCIENCE (MATHEMTICS & COMPUTER SCIENCE)

SMA 407: MEASURE THEORY

DATE: 22/7/2019 **TIME: 2:00 – 4:00 PM**

INSTRUCTIONS:

(a) Answer <u>ALL</u> the questions in Section A and <u>ANY TWO</u> Questions in Section B

SECTION A

QUESTION ONE 30 MARKS

a) Define a measure? (3 marks)

b) Prove that if μ is a measure defined on a σ −algebra $\mathfrak{X}$. Then μ is monotonic, that is if $E \subset F$, then $\mu(E) \leq \mu(F)$. Furthermore if $\mu(E) \leq \infty$, then $\mu(F - E) = \mu(F) - \mu(E)$.
. (5 marks)

c) Prove that $\mu^*(\{x\}) = 0$ for all $x \in \mathbb{R}$ (4 marks)

d) Show that if $\mu^*(E) = 0$, then E is L-measurable. (5 marks)

e) Prove that the space $(\mathbb{R}, \mathcal{M}, \mu)$, where μ is the lebesque measure is complete. (5 marks)

f) Prove that if $f, g: \to \mathbb{R}$ are two $\mathfrak{X}$ −measurable functions and c be a real number then the function cf is $\mathfrak{X}$ −measurable (5 marks)

g) Prove that if φ and ρ are simple functions in $M^+(X, \mathfrak{X})$ and $c \geq 0$, then $\int c\varphi du = c \int \varphi du$ (3 marks)

SECTION B

QUESTION TWO 20 MARKS

a) Prove that if $f, g: \rightarrow \mathbb{R}$ are two $\mathfrak{X}$ −measurable functions and c be a real number then the function

i) f^2

ii) $|f|$ are $\mathfrak{X}$ −measurable. (@5 marks)

b) Let (f_n) be a sequence of $\mathfrak{X}$ −measurable functions $f_n: X \rightarrow \mathbb{R}$, then the functions $f_1 \cup f_2 \cup f_{3\cup} \ldots \ldots \ldots \cup f_n$ and $f_1 \cap f_2 \cap f_3 \cap \ldots \ldots \ldots \cap f_n$ is $\mathfrak{X}$ −measurable. (@5 Mrk)

QUESTION THREE 20 MARKS

a) State and prove monotone convergence theorem. (10 marks)

b) Let $(X, \mathfrak{X})$ be a measurable space. Then if $f, g \in M^+(X, \mathfrak{X})$, then

i) $f + g \in M^+(X, \mathfrak{X})$ (6 marks)

ii) $\int cfdu = c \int fdu$ (4 marks)

QUESTION FOUR 20 MARKS

a) Prove that if f and g both belong to M^+ and $f \leq g$, then $\int fdu \leq \int gdu$ (6 marks)

b) Prove that if $f \in M^+$ and if $E, F \in \mathfrak{X}$, with $E \complement F$ then $\int_E \; fdu \leq \int_F \; fdu$ (7 marks)

c) State and prove Fatous lemma. (7 marks)

QUESTION FIVE 20 MARKS

a) Define σ −algebra (5 marks)

b) Prove that if $A \complement B$ then $\mu^*(A) \leq \mu^*(B)$, $A, B \in \mathbb{R}$ (5 marks)

c) Prove that $\mu^*(\emptyset) = 0$ (3 marks)

d) Prove that lebesque outer measure μ^* is countably sub additive. (7 marks)