MACHAKOS UNIVERSITY

ISO 9001:2008 Certified

School of Pure and Applied Sciences, Education (Arts and Science) Examination

Partial Differential Equation II SMA 432

Fourth year Supplementary Examinations 2018

Mathematics and Statistics.

Time 2hours

Attempt question ***one (compulsory)*** and any other ***two questions***.

Question 1(30mks)

a) Solve the differential equation $\frac{dx}{\cos(x+y)} = \frac{dy}{\sin(x+y)} = \frac{dz}{z}$ (5mks)

b) Determine the orthogonal trajectories on the conicoid $(x+y)z = 1$ of the conics in which it is cut by the system of planes $x - y + z = k$, where k is a parameter. (5mks)

c) Solve the Lagrange's equation $y^2 p - xyq = x(z - 2y)$ (5mks)

d) Work out the characteristics and the corresponding transport equations of the system

$$xyu_x - v_y = 0$$
$$xu_y - v_x = 0$$
(5mks)

e) Determine the complete integral of $z = px + qy + p^2 + q^2$ using Charpit's method (5mks)

f) Eliminate the arbitrary function f from the equation $f\left(x^2 + y^2 + z^2, z^2 - 2xy\right) = 0$ (5mks)

Question 2(20mks)

a) Using Jacobi's method determine a complete integral of $p_1 x_1 + p_2 x_2 = p_3^2$

(10mks)

b) Consider the partial differential equation $p^2 + q^2 - 2pq \tanh 2y = \sec h^2 2y$;

i.) Write its Charpit's auxiliary equations (2mks)

ii.) Solve the p.d.e by considering its Charpit's auxiliary equations. (8mks)

Question 3(20mks)

a) Solve ;

i.) $\frac{dx}{x^2}=\frac{dy}{y^2}=\frac{dz}{nxy}$ (7mks)

ii.) $\frac{dx}{y^2(x-y)}=\frac{dy}{-x^2(x-y)}=\frac{dz}{z(x^2-y^2)}$ (8mks)

iii.) Show that $(2x+y^2+2xz)dx+2xydy+x^2dz=0$ is integrable (5mks)

Question 4(20mks)

a) Determine the orthogonal trajectories on the cone $x^2+y^2=z^2\tan^2\alpha$ of its intersection with the family of planes parallel to $z=c$

(8mks)

b) Solve the semi-linear equation $x^2\frac{\partial z}{\partial x}+y^2\frac{\partial z}{\partial y}=(x+y)z$ (7mks)

c) Consider the partial differential equation $z = px+qy+p+q-pq$ with a complete integral of the form $z=ax+by+p+a+b-ab$; where a and b are arbitrary constants. Determine a general solution by finding the envelope of those planes that pass through the origin. (5mks)

Question 5(20mks)

a) Calculate the integral surface of the quasi-linear partial differential equation $x(y^2+z)p-y(x^2+z)q=(x^2-y^2z)$ which contains the straight line $x+y=0$, $z=1$

(8mks)

b) in the one-dimensional unsteady flow of compressible fluid, the velocity u and c where $c^2=\frac{dp}{d\rho}$, p is pressure and ρ is density satisfying the following equations.

$$\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial x}+2c\frac{\partial c}{\partial x}=0$$

$$2\frac{\partial c}{\partial t}+2u\frac{\partial c}{\partial x}+c\frac{\partial u}{\partial x}=0$$

Prove that the characteristics are given by the differential equation $dx=(u\pm c)dt$ and that on the characteristic $dx=(u+c)dt$, $u+2c$ is a constant. (12mks)