MACHAKOS UNIVERSITY

University Examinations 2018/2019

SCHOOL OF PURE AND APPLIED SCIENCES

DEPARTMENT OF PHYSICAL SCIENCES

SECOND YEAR SUPPLEMENTARY EXAMINATION FOR
BACHELOR OF EDUCATION (SPECIAL NEEDS)
BACHELOR OF SCIENCE (MATHEMATICS)
BACHELOR OF EDUCATION (SCIENCE),

SPH 200: MECHANICS II

DATE: 27/9/2019 **TIME: 8:30 – 10:30 AM**

Important Constants

$g = 9.81\ N/kg$

Instructions to Candidates

Answer *Question one and any other two* questions from this paper.

Cheating will lead to automatic disqualification.

QUESTION ONE (30 MARKS)

a) (i) Define torque and state its mathematical expression. (2 marks)

(ii) Considering the Newton's second law of motion, $\vec{\mathrm{F}} = \frac{\mathrm{d}\vec{\mathrm{P}}}{\mathrm{dt}}$ where $\vec{P}$ is linear momentum and $\vec{\mathrm{F}}$ is the resultant force, derive the expression of the law of conservation of angular momentum $\overrightarrow{L}$. (5 marks)

b) (i) What is a conservative force? (2 marks)

(ii) Mention one example of a conservative force and one non-conservative force. (2 marks)

c) Show that the force field $\vec{\mathrm{F}}$ defined by,

$$\boldsymbol{F} = x^2\mathbf{i} + 2yz\mathbf{j} + y^2\mathbf{k}$$

is a conservative force field. (3 marks)

Hence compute the scalar potential function V that generates $\vec{\mathrm{F}}$ at the point (1,0,3) (5 marks)

d) State and give expressions of **two** physical quantities that are expressed as vector products. (4 marks)

e) Determine the moment of inertia of a circular disc of radius R and mass M rotating about its own axis. Hence use the perpendicular axis theorem to determine the moment of inertia about its diameter. (4 marks)

f) Given two vectors $\vec{A} = A_1\boldsymbol{i} + A_2\boldsymbol{j} + A_3\boldsymbol{k}$ and $\vec{B} = B_1\boldsymbol{i} + B_2\boldsymbol{j} + B_3\boldsymbol{k}$
show that $\vec{A} \times \vec{B} = (A_2B_3 - A_3B_2)\boldsymbol{i} + (A_3B_1 - A_1B_3)\boldsymbol{j} + (A_1B_2 - A_2B_1)\boldsymbol{k}$ (3 marks)

QUESTION TWO (20 MARKS)

a) (i) State the theorem of parallel axes as applied to moment of inertia. (2 marks)
(ii) Derive the expression used in the parallel axis theorem of a rigid body showing clearly the axes of rotation. (4 marks)

b) (i) What is *center of mass* of a system of particles? (2 marks)
(ii) Two particles of masses $m_a = 1.2\ kg$ and $m_b = 0.3\ kg$ are at distance of 0.9 m apart. Calculate the moment of inertia of the system about an axis through the center of mass and perpendicular to the line joining the two masses.
(6 marks)

c) Determine the moment of inertia of a thin uniform rod of mass M and length L rotating about an axis through its center perpendicular to its length. (6 marks)

QUESTION THREE (20 MARKS)

a) (i) Distinguish between a *damped harmonic motion* and *simple harmonic motion*.
(2 marks)
(ii) State and explain one application of *damping* in mechanical systems. (2 marks)

b) A 40g mass is attached to the lower end of a spring of negligible mass and oscillates at a frequency of 2 Hz , with an amplitude of 8 cm
i) Determine the angular velocity. (3 marks)
ii) How fast is the system moving when it is 2 cm from the equilibrium?
(3 marks)
iii) What is the maximum kinetic energy of this mass? (4 marks)

c) If the amplitude of a simple harmonic oscillator is doubled, how does this affect,
i) its periodic time, (2 marks)
ii) its total energy, and (2 marks)
iii) the maximum velocity of the oscillator. (2 marks)

QUESTION FOUR (20 MARKS)

a) (i) What is *moment of inertia* of a body? (2 marks)
(ii) Describe the perpendicular axis theorem as used in rotational motion. (2 marks)

b) A horizontal disc rotating freely about a vertical axis makes 100 revolutions per minute. A small piece of wax of mass 10g falls vertically on to the disc and adheres to it at a distance of 9 cm from the axis. If the number of revolutions per minute is thereby reduced to 90 cm, calculate the moment of inertia of the disc. (5 marks)

c) A circular disc of mass m and radius r rolls on a table. If ω is its angular speed. Show that its total energy is given by $E = \frac{3}{4}r^2\omega^2$. (4 marks)

d) Given that
$\vec{A} = 2\boldsymbol{i} + 3\boldsymbol{j} - 5\boldsymbol{k}$ and $B^{\rightarrow} = 4\boldsymbol{i} + \boldsymbol{j} + 2\boldsymbol{k}$
Calculate,
i) The angle between the two vectors. (3 marks)
ii) The unit vector perpendicular to the plane containing the two vectors (4 marks)

QUESTION FIVE (20 MARKS)

a) (i) What is radius of gyration in moment of inertia? (2 marks)

(ii) Calculate the radius of gyration of a square lamina of mass m and side 9cm rotating about an axis along one of the edges. (4 mark)

b) Three equal uniform rods each of length l and mass m are rigidly jointed to form an equilateral triangle as shown in Figure 1. Derive the expression for the moment of inertia of the figure rotating about an axis through one vertex perpendicular to the plane of the triangle. (8 marks)

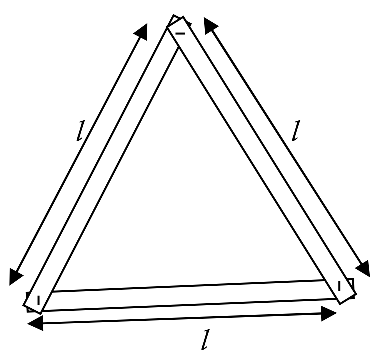

Figure 1

c) A flexible rope is wrapped several times around a solid cylinder of diameter 0.12 m which rotates without friction about a fixed horizontal axis with moment of inertia of $0.9\ \text{kg.m}^2$. The free end of the rope is pulled with a constant force of 9 N for a distance of 0.75 m. The cylinder is initially at rest. Calculate the final angular velocity and the final speed of the rope. (5 marks)