

SPH 400: CLASSICAL MECHANICS

DATE: SCHOOL BASED

TIME: 2 Hours

INSTRUCTIONS

Answer **<u>QUESTION ONE</u>** which COMPULSORY and ANY OTHER TWO questions.

Question 1 carries **30** marks and the rest **20** marks each.

YOU MAY USE:

You may need to use the following constants

- Earth's gravitational acceleration = 9.8 N/Kg
- The Universal Gravitational Constant G = $6.67 \times 10^{-11} \text{ N.m}^2/\text{Kg}^2$
- Permeability of free space, $\mu o=4\pi \times 10-7$ Tm/A

QUESTION ONE (COMPULSORY) (30 MARKS)

- a) State the three laws of classical motion (3 marks)
- b) Identify the two assumptions made under Galilean transformations.
- c) Find the number of degrees of freedom for a rigid body which has one point fixed but can move in space about this point.
 (3 marks)
- d) A system of particles consists of particles of mass 3g located at point P (1, 0, -1), 5g at point Q (-2, 1, 3) and 2g at point R (3, -1, 1). Find the coordinates of the center of mass of the system.
- e) Describe the three constraints on a system that are involved in constrained motion. (6 marks)

(2 marks)

- f) A 600kg rocket is set for vertical firing. If the exhaust speed is 1000ms⁻¹, how much gas must be ejected each second to supply the thrust needed
 - i) to overcome the weight of the rocket (4 marks)
 - ii) to give the rocket an initial upward acceleration of 20ms⁻²? (3 marks)
- g) Consider a body of mass m be subjected to a force making an angle θ with the direction of motion, show that $F \cos \theta \mu_k (mg F \sin \theta) = 0$ when there's a *d* displacement. (5 marks)



QUESTION TWO (20 MARKS)

a) Due to a force field, a particle of mass 5 units moves along a space curve whose position

vector is given as a function of time t by $\vec{r} = (2t^3 + t)\hat{i} + (3t^4 - t^2 + 8)\hat{j} - 12t^2\hat{k}$. Find

- (i) the velocity (2 marks)
- (ii) the momentum (2 marks)
- (iii) the acceleration (2 marks)
- (iv) the force at any time t. (2 marks)
- b) An alpha particle, which is the nucleus of the He-atom, is emitted from a Uranium-238 nucleus, originally at rest, with a speed of $1.4 \times 10^7 \text{ms}^{-1}$ and a kinetic energy of 4.1 MeV. Find the recoil speed of the residual nucleus which is Thorium-234. (6 marks)
- c) A rocket is moving away from the solar system at a speed of $6.0 \times 10^3 \text{ms}^{-1}$. It fires its engine which ejects exhaust with a relative velocity of $3.0 \times 10^3 \text{ms}^{-1}$. The mass of the rocket at this time is $4.0 \times 10^4 \text{kg}$ and it experiences an acceleration of 2.0ms^{-2} . Find;
 - i) the velocity of the exhaust relative to the solar system (3 marks)
 - ii) the rate at which the exhaust was ejected during the firing. (3 marks)

QUESTION THREE (20 MARKS)

- a) Show that the theory of Parallel axis is $I = I_{CM} + Mh^2$. Where all the terms have their usual meaning (10 marks)
- b) Use Hamilton's Principle to find the equation of motion of a one-dimensional harmonic oscillator. (10 marks)

QUESTION FOUR (20 MARKS)

Two skaters collide and embrace each other. One has mass $m_1 = 70$ kg and is initially moving east at a speed $u_1 = 6$ km/h while the other has mass $m_2 = 50$ kg and is initially moving north at a speed $u_2 = 8$ km/h.

- (a) What is the final velocity of the couple? (10 marks)
- (b) What fraction of the initial kinetic energy is lost because of the collision? (10 marks)

QUESTION FIVE (20 MARKS)

- a) Consider a system of many particles. Show that under rotational motion, $\vec{L} = \vec{R} \times M v_{CM} + \sum_{i} \vec{r}'_{i} \times \vec{p}'_{i}$ where all the terms have their usual meaning. (10 marks) b) Consider a mass *m* be attached to a spring of force constant *k*. Obtain its equation of motion
- b) Consider a mass m be attached to a spring of force constant k. Obtain its equation of motion and the frequency of oscillation using Lagrange equations. (10 marks)