MACHAKOS UNIVERSITY

SCHOOL OF PURE AND APPLIED SCIENCES

DEPARTMENT OF MATHEMATICS AND STATISTICS

FIRST YEAR, SECOND SEMESTER EXAMINATIONS

For Diploma in Mechanical Engineering. (TVET)

Date Time **3 hours**

Instructions

The paper consists of **EIGHT** questions. Answer any **FIVE** questions.

ALL questions carry equal marks.

Show all your working

1. (a) Simplify the expressions;

   i) $\frac{(1-x)^{\frac{1}{2}}-x(1-x)^{-\frac{1}{2}}}{1-x}$

   ii) $\frac{log729-4log3+2log27}{log243-log27+log9}$ without using logarithm tables (7marks)

   b) Solve the equations;

   i) $\log_2 x + 2\log_4(x+1) = 1$

   ii) $4^x = 2 + 16^{\frac{x}{4}}$ (13 marks)

2. a) Determine the values of p, q, and r such that $4x^2 - 3x + 12 = p(x + q)^2 + r$ (5 marks)

   b) The roots of the equation $ax^2 + bx + c = 0$ are α *and* $\alpha + 2$. Prove that

   $b^2 = 4(a^2 + ac)$. (7marks)

   c) Solve the following simultaneous equations

   $x + 2y - z = 1$

   $x + 3y - 2z = 0$

   $x + y + z = 4$

Use the method of substitution to solve the equation (8 marks)

3) a) Simplify the expression $5 \times 4^{3n+1} - 20 \times 8^{2n}$ (4 marks)

b) Find the values of:

i) $\frac{log15625}{log25} - 2$

ii) $\frac{8^{\frac{2}{3}}+4^{\frac{3}{2}}}{16^{\frac{3}{4}}}$ (6 marks)

c) Given that 2log8N = p, log22N = q and that q – p = 4, determine the value of N.

(10 marks)

4.a) Given that SinA = $\frac{12}{13}$ and Cos B = $\frac{4}{5}$ where A is obtuse and B is acute, determine the values of ;

i) Sin (A – B)

ii) Tan (A + B) (5 marks)

b) Prove the identities:

i) $\frac{1-Cos\theta}{Sin\theta} + \frac{Sin\theta}{1-Cos\theta} = 2\text{Cosec}\theta$

ii) tan 3x = $\frac{3tanx - tan^3x}{1-3tan^2x}$ (8 marks)

c) Given t = $tan22\frac{1}{2}^0$

i) Show that $\tan 45^0 = \frac{2t}{1-t^2}$;

ii) Hence solve the equation:

$t^2 + 2t - 1 = 0$, leaving your answer in surd form. (7 marks)

5.a) Express in polar co-ordinates the position :

i) P_1 (3 4) ii) P_2(-5 -8) (6 marks)

b) obtain the Cartesian equations of;

i) r = 5(1 + 2cosθ)

ii) r = a tanθ (7 marks)

c) Find the cartesian equations of the loci;

i) $x = t^2 + 4$ and $y = t - 3$Type equation here.

ii) $x = 5\cos\theta$ and $y = 4\sin\theta$ (7 marks)

6.a) Obtain the (i) polar equation of the of the loci $x^2 + y^2 - 2x = 0$

(ii) parametric equation of the locus $x^3 + y^3 = 3xy$ (7 marks)

b) Express $\frac{6-7j}{j(5-2j)} + \frac{3}{2j}$ (5 marks)

c) Solve for $0^0 \leq x \leq 360^0$ given;

$12\sin2x + 12\cos x - 6\sin x - 3 = 0$ (8 marks)

7.a) Given the complex numbers $Z_1 = 4 + 3j$, $Z_2 = 1 + j$ and $Z_3 = 1 - 2j$, express

$Z = Z_1 + \frac{Z_2Z_3}{Z_2+Z_3}$ in the form a + bj. (8 marks)

b) Use De Moivre's theorem to prove that $\cos 3\theta = 4Cos^3\theta - 3Cos\theta$ (5 marks)

c) Given that Z = j is one root of the equation $Z^4 - 2Z^3 + 3Z^2 - 2Z + 2 = 0$,

determine the other roots. (7 marks)

8.a) solve the following equations for all values of θ between 0^0 and 360^0.

$2\sin\theta - 3cos\theta = 2$ (7 marks)

b) Solve for x in the following equations

i) $\log_3(2x-3) = -1$ (3 marks)

ii) $3^{2x} = 4(3^x) + 3$ (4 marks)

c) Show that $\frac{1+tan^2B}{1+cot^2B} = tan^2B$ (6 marks)