MACHAKOS UNIVERSITY

SCHOOL OF PURE AND APPLIED SCIENCES

DEPARTMENT OF MATHEMATICS AND STATISTICS

FIRST YEAR, SECOND SEMESTER EXAMINATIONS

For Diploma in Electrical and Electronics. (TVET)

Date Time **3 hours**

Instructions

The paper consists of **EIGHT** questions. Answer any **FIVE** questions.

ALL questions carry equal marks.

Show all your working

1. (a) Simplify the expressions;

 i) $\frac{(1-x)^{\frac{1}{2}}-x(1-x)^{-\frac{1}{2}}}{1-x}$

 ii) $\frac{log729-4log3+2log27}{log243-log27+log9}$ without using logarithm tables (7marks)

 b) Solve the equations;

 i) $\log_2 x + 2\log_4(x+1) = 1$

 ii) $4^x = 2+ 16^{\frac{x}{4}}$

 (13 marks)

2. a) Determine the values of p, q, and r such that $4x^2 - 3x + 12 = p(x + q)^2 + r$ (5 marks)

 b) The roots of the equation $ax^2 + bx + c = 0$ are α and $\alpha + 2$. Prove that

 $b^2 = 4(a^2 + ac)$. (7marks)

 c) Three currents in a d.c. circuit satisfy the simultaneous equations

 $I_1 + 2I_2 - I_3 = 1$

 $I_1 + 3I_2 - 2I_3 = 0$

 $I_1 + I_2 + I_3 = 4$

Use the method of substitution to solve the equation (8 marks)

3) a) Simplify the expression $5 \times 4^{3n+1} - 20 \times 8^{2n}$ (4 marks)

b) Find the values of:

i) $\frac{log15625}{log25} - 2$

ii) $\frac{8^{\frac{2}{3}}+4^{\frac{3}{2}}}{16^{\frac{3}{4}}}$ (6 marks)

c) Given that 2log8N = p, log22N = q and that q – p = 4, determine the value of N.

(10 marks)

4.a) Given that SinA = $\frac{12}{13}$ and Cos B = $\frac{4}{5}$ where A is obtuse and B is acute, determine the values of ;

i) Sin (A – B)
ii) Tan (A + B)

b) Prove the identities:

i) $\frac{1-Cos\theta}{Sin\theta} + \frac{Sin\theta}{1-Cos\theta} = 2\text{Cosec}\theta$

ii) tan 3x = $\frac{3tanx - tan^3x}{1-3tan^2x}$ (8 marks)

c) Given t = $tan22\frac{1}{2}^0$

i) Show that $\tan45^0 = \frac{2t}{1-t^2}$;

ii) Hence solve the equation:

$t^2 + 2t - 1 = 0$, leaving your answer in surd form. (7 marks)

5.a) Express in polar co-ordinates the position :

i) P_1 (3 4) ii) P_2(-5 -8) (6 marks)

b) Obtain the Cartesian equations of;

i) r = 5(1 + 2cosθ)

ii) r = a tanθ (7 marks)

c) Find the cartesian equations of the loci;

i) $x = t^2 + 4$ and $y = t - 3$

ii) $x = 5\cos\theta$ and $y = 4\sin\theta$ (7 marks)

6.a) solve the following equations for all values of θ between 0^0 and 360^0.

$2\sin\theta - 3\cos\theta = 2$ (7 marks)

b) Solve for x in the following equations

i) $\log_3(2x-3) = -1$ (3 marks)

ii) $3^{2x} = 4(3^x) + 3$ (4 marks)

c) Show that $\frac{1+tan^2B}{1+cot^2B} = tan^2B$ (6 marks)

7.a) Given that $p\cosh x + q\sinh x = 3e^x - 2e^{-x}$, determine the values of p and q. (7 marks)

b) Prove the identities:

i) $\cosh 2x = \frac{1+tanh^2x}{1-tanh^2x}$

ii) $\tanh 3x = \frac{3tanhx+tanh^3x}{1+3tanh^2x}$ (6 marks)

c) Solve the equation:

$3\cosh x - 7\sinh x = 2$ (7 marks)

8.a) Given the complex numbers $Z_1 = 4 + 3j$, $Z_2 = 1 + j$ and $Z_3 = 1 - 2j$, express

$Z = Z_1 + \frac{Z_2Z_3}{Z_2+Z_3}$ in the form a + bj. (8 marks)

b) Use De Moivre's theorem to prove that $\cos 3\theta = 4Cos^3\theta - 3Cos\theta$ (5 marks)

c) Given that Z = j is one root of the equation $Z^4 - 2Z^3 + 3Z^2 - 2Z + 2 = 0$,

determine the other roots. (7 marks)