MACHAKOS UNIVERSITY

University Supplementary Examinations

SMA 331: DYNAMICS II

INSTRUCTIONS TO CANDIDATES

Answer question **ONE** (**compulsory**) and any other **TWO** questions

Question one (30 Marks)

a. State some important properties of a central force fields **(3 Marks)**
b. Prove that if a particle moves in a central force field, then its path must be a plane curve **(5 Marks)**
c. Given $m(\boldsymbol{r}\ddot{\theta} + 2\dot{\boldsymbol{r}}\dot{\theta}) = 0$, show that the equation of motion for a particle in a central field is $\boldsymbol{r}^2\dot{\theta} = h$, a constant **(4 Marks)**
d. Show by means of substitution $\boldsymbol{r} = \frac{1}{u}$ that the differential equation for the path of the particle in a central field is $\frac{d^2u}{d\theta^2} + u = -\frac{f\left(\frac{1}{u}\right)}{mh^2u^2}$ **(8 Marks)**
e. Calculate the angular speed of the earth about its axis **(4 Marks)**
f. Two particles of masses m_1 and m_2 respectively are connected by a rigid massless rod of length, a and move freely in a plane. Show that the moment of inertia of the system about an axis perpendicular to the plane and passing through the center of mass is μa^2 where the reduced mass $\mu = \frac{m_1m_2}{(m_1+m_2)}$ **(6 Marks)**

Question two (20 Marks)

a. Using the calculus of variations and the Hamilton's principle, prove that a necessary condition for $\boldsymbol{I} = \int_{\boldsymbol{a}}^{\boldsymbol{b}} \boldsymbol{F}(x, y, y')dx$ to be an extremum (maximum or minimum) is $\frac{d}{dx}\left(\frac{\partial \boldsymbol{F}}{\partial y'}\right) - \frac{\partial \boldsymbol{F}}{\partial y} = 0$ **(5 Marks)**

b. Obtain the equation of motion $\ddot{\boldsymbol{\theta}} + \frac{Mga}{I_0}\sin\boldsymbol{\theta}$ for a compound pendulum where the force $Mg = -Mg\boldsymbol{j}$ is conservative so that the potential energy $\boldsymbol{V}$ is such that $-\nabla\boldsymbol{V} = -\frac{\partial \boldsymbol{V}}{\partial x}\boldsymbol{i} - \frac{\partial \boldsymbol{V}}{\partial y}\boldsymbol{j} - \frac{\partial \boldsymbol{V}}{\partial z}\boldsymbol{k}$ **(5 Marks)**

c. Find the moment of inertia of a right circular cone of height, h and radius, a about its axis and the radius of gyration, K using triple integration given

$$I = \int_{\theta=0}^{2\pi}\int_{r=0}^{a}\int_{z=0}^{\frac{h(a-r)}{a}} \sigma r^3 dr d\theta dz$$

and

$$M = \int_{\theta=0}^{2\pi}\int_{r=0}^{a}\int_{z=0}^{\frac{h(a-r)}{a}} \sigma r dr d\theta dz$$

(10 Marks)

Question three (20 Marks)

An xyz coordinate system is rotating with respect to an XYZ coordinate system having the same origin and assumed to be fixed in space i.e. it is an inertial system. The angular velocity of the xyz system relative to the XYZ system is given by $\boldsymbol{\omega} = 2t\boldsymbol{i} - t^2\boldsymbol{j} + (2t+4)\boldsymbol{k}$ where t is the time. The position vector of a particle at time t as observed in the xyz system is given by $\boldsymbol{r} = (t^2+1)\boldsymbol{i} - 6t\boldsymbol{j} + 4t^3\boldsymbol{k}$ at time $t = 1$, find the magnitude of the following quantities:-

i. The apparent velocity given by $\frac{d\boldsymbol{r}}{dt}$ **(2 Marks)**

ii. The true velocity given by $\frac{d\boldsymbol{r}}{dt} + \boldsymbol{\omega} X \boldsymbol{r}$ **(5 Marks)**

iii. the apparent acceleration given by $\frac{d^2\boldsymbol{r}}{dt^2}$ **(2 Marks)**

iv. The true acceleration of the particle given by

$$\frac{d^2\boldsymbol{r}}{dt^2} + 2\boldsymbol{\omega}X\frac{dr}{dt} + \frac{d\omega}{dt}X\boldsymbol{r} + \boldsymbol{\omega}X(\boldsymbol{\omega}X\boldsymbol{r})$$ **(6 Marks)**

v. The Coriolis acceleration given by, $2\boldsymbol{\omega}X\frac{dr}{dt}$ **(5 Marks)**

Question four (20 Marks)

a. Prove that $\frac{\partial \dot{r}_v}{\partial \dot{q}_\alpha} = \frac{\partial r_v}{\partial q_\alpha}$ where $\boldsymbol{r}_v = \boldsymbol{r}_v(q_1, q_2, \dots\dots\dots. q_n, t)$ **(3 Marks)**

b. Let $\boldsymbol{F}_V$ be the net external force acting on the v^{th}particle of a system. Prove that

$$\frac{d}{dt}\left\{\sum_v m_v\dot{\boldsymbol{r}}_v.\frac{\partial \boldsymbol{r}_v}{\partial q_\alpha}\right\} - \sum_v m_v\dot{\boldsymbol{r}}_v.\frac{\partial \dot{\boldsymbol{r}}_v}{\partial q_\alpha} = \sum_v \boldsymbol{F}_v.\frac{\partial \boldsymbol{r}_v}{\partial q_\alpha}$$

(6 Marks)

c. Prove the Lagrange's equations,

$$\frac{d}{dt}\left(\frac{\partial T}{\partial \dot{q}_\alpha}\right) - \frac{\partial T}{\partial q_\alpha} = \phi_\alpha, \alpha = 1, 2, \dots\dots\dots\dots\dots\dots. n$$

Where

$$\phi_\alpha = \sum_v \boldsymbol{F}_v.\frac{\partial \boldsymbol{r}_v}{\partial q_\alpha}$$

(5 Marks)

d. If the Hamiltonian $\boldsymbol{H} = \sum p_\alpha \dot{q}_\alpha - L$, where the summation extends from $\alpha = 1$ to n, is expressed as a function of the coordinates, q_α and moments, p_α prove the Hamilton's equations $\dot{p}_\alpha = -\frac{\partial \boldsymbol{H}}{\partial q_\alpha}$ and $\dot{q}_\alpha = \frac{\partial \boldsymbol{H}}{\partial p_\alpha}$ regardless whether $\boldsymbol{H}$

i. Does not contain the variable time, t explicitly **(3 Marks)**

ii. Does contain the variable time, t explicitly **(3 Marks)**

Question five (20 Marks)

a. Using the calculus of variations and the Hamilton's principle, prove that a necessary condition for $\boldsymbol{I} = \int_{\boldsymbol{a}}^{\boldsymbol{b}} \boldsymbol{F}(x, y, y')dx$ to be an extremum (maximum or minimum) is $\frac{d}{dx}\left(\frac{\partial \boldsymbol{F}}{\partial y'}\right) - \frac{\partial \boldsymbol{F}}{\partial y} = 0$ **(5 Marks)**

b. Obtain the equation of motion $\ddot{\boldsymbol{\theta}} + \frac{Mga}{I_0}\sin\boldsymbol{\theta}$ for a compound pendulum where the force $Mg = -Mg\boldsymbol{j}$ is conservative so that the potential energy $\boldsymbol{V}$ is such that $-\nabla\boldsymbol{V} = -\frac{\partial \boldsymbol{V}}{\partial x}\boldsymbol{i} - \frac{\partial \boldsymbol{V}}{\partial y}\boldsymbol{j} - \frac{\partial \boldsymbol{V}}{\partial z}\boldsymbol{k}$ **(5 Marks)**

c. Find the moment of inertia of a right circular cone of height, h and radius, a about its axis and the radius of gyration, K using triple integration given

$$I = \int_{\theta=0}^{2\pi}\int_{r=0}^{a}\int_{z=0}^{\frac{h(a-r)}{a}} \sigma r^3 dr d\theta dz$$

and

$$M = \int_{\theta=0}^{2\pi}\int_{r=0}^{a}\int_{z=0}^{\frac{h(a-r)}{a}} \sigma r dr d\theta dz$$

(10 Marks)

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