MACHAKOS UNIVERSITY

University Examinations for 2022/2023 Academic Year

SCHOOL OF PURE AND APPLIED SCIENCES

DEPARTMENT OF MATHEMATICS AND STATISTICS

THIRD YEAR FIRST SEMESTER EXAMINATIONS FOR

BACHELOR OF SCIENCE (ELECTRICAL AND ELECTRONICS ENGINEERING)

BACHELOR OF SCIENCE (CIVIL ENGINEERING)

BACHELOR OF SCIENCE (MECHANICAL ENGINEERING)

ECU 300: ENGINEERING MATHEMATICS IX

DATE: **TIME:**

INSTRUCTIONS

Answer question **one** (**Compulsory**) and any other **two** questions.

QUESTION ONE (COMPULSORY) (30 MARKS)

a) Find unit tangent vector to the curve $x = 2t^2 - 6t, y = 4t - 3, z = t^2 + 1$ at $t = 2$ (5 marks)

b) Find $\nabla\left(\frac{1}{r}\right)$ where $r = (x^2 + y^2 + z^2)^{\frac{1}{2}}$ and $\vec{r} = x\hat{\imath} + y\hat{\jmath} + z\hat{k}$ (5 marks)

c) Find the stationary points of $f(x, y) = xy^2 - x^2y^2 + x^4 + 3$ (5 marks)

d) Determine the unit vectors in the directions of the following vectors and determine whether the vectors are orthogonal:

$2\boldsymbol{i} + 3\boldsymbol{j} - \boldsymbol{k}, 4\boldsymbol{i} - 2\boldsymbol{j} + 2\boldsymbol{k}, \boldsymbol{i} + 4\boldsymbol{j} + 2\boldsymbol{k}$ (5 marks)

e) Evaluate $\int_0^{\frac{\pi}{2}} \int_0^{\frac{\pi}{4}} \sin(x + y)\, dxdy$ (5 marks)

f) Using Green's theorem, evaluate $\oint y^3 dx - x^3 dy$ taken over curve C which is a positively oriented circle radius 2 centred at the origin. (5 marks)

QUESTION TWO (20 MARKS)

a) Given that $\vec{U} = (2t-3)\boldsymbol{i} + \boldsymbol{j} - t\boldsymbol{k}$ and $\vec{V} = t^2\boldsymbol{i} - t\boldsymbol{j} + (2t+1)\boldsymbol{k}$ determine;

i) $\left|\frac{d}{dt}\left(\vec{U}(t) \cdot \vec{V}(t)\right)\right|$ at $t = 1$ (5 marks)

ii) $\left|\frac{d}{dt}\left(\vec{U}(t) \times \vec{V}(t)\right)\right|$ at $t = 1$ (5 marks)

b) If $x = t - \frac{t^3}{3}, y = t^2 \; and \; z = t + \frac{t^3}{3}$, determine;

i) The unit tangent vector $\hat{T}$ (5 marks)

ii) The curvature k (5 marks)

QUESTION THREE (20 MARKS)

a) Find the work done by the force field $F(x,y,z) = -\frac{1}{2}x\boldsymbol{i} - \frac{1}{2}y\boldsymbol{j} + \frac{1}{4}\boldsymbol{k}$ as it moves along $\vec{r}(t) = \cos \boldsymbol{t}\, i + \sin \boldsymbol{t}\, j + t\, \boldsymbol{k}$ from point $(1,0,0)$ to $(1,0,3\pi)$ (5 marks)

b) Given $z = x^2y + xy^2, x = 2 + t^4, y = 1 - t^3$, find $\frac{dz}{dt}$ (5 marks)

c) Evaluate $\iint \vec{A} \cdot \hat{n} dS$ over surface S where $\vec{A} = z\boldsymbol{i} + x\boldsymbol{j} + 3y^2z\boldsymbol{k}$ where S is the surface of the cylinder $x^2 + y^2 = 16$ included in the yz plane between $z = 0$ and $z = 5$ (10 marks)

QUESTION FOUR (20 MARKS)

a) Determine whether the vector field $\vec{F} = (3x^2y^2z + 5y^3, 2x^3yz + 15xy^2 - 7z, x^3y^2 - 7y + 4z^3)$ is conservative and if it is find its scalar potential function (10 marks)

b) Show that the cylindrical coordinate system is orthogonal. (5 marks)

c) Prove that $div\left(Curl\,\vec{V}\right) = 0$ where $\vec{V} = v_1\boldsymbol{i} + v_2\boldsymbol{j} + v_3\boldsymbol{k}$ (5 marks)

QUESTION FIVE (20 MARKS)

a) A fluid motion is given by $\vec{V} = (y+z)\boldsymbol{i} + (z+x)\boldsymbol{j} + (x+y)\boldsymbol{k}$,

i. Show that this motion is irrotational. (3 marks)

ii. Find the velocity potential (7 marks)

iii. Show that this motion is possible for incompressible fluid (2 marks)

b) Use Stoke's theorem to evaluate $\oint x^2dx + xydy$ over the square whose sides are $x = 0, y = 0, x = a, y = a$ in the plane $z = 0$ (8 marks)