



MACHAKOS UNIVERSITY

University Examinations 2020/2021 Academic Year

SCHOOL OF PURE AND APPLIED SCIENCES

DEPARTMENT OF MATHEMATICS AND STATISTICS

SECTION A: ANSWER ALL QUESTIONS IN THIS SECTION

QUESTION ONE (30 MARKS)

a) Define the following terms: [4 Mks]

i) Set

ii) Define cardinality of a set and give its symbolic notation

b) Evaluate the following expressions.

i)

$$\frac{(3+9 \times 6) \div 3 - 2 \div 2}{3 \times 6 + (4-9) - 3^2 + 5}$$
 [3 Mks]

ii)

$$\frac{1}{3} \text{ of } \left(5\frac{1}{2} - 3\frac{3}{4} \right) + 3\frac{1}{5} \div \frac{4}{5} - \frac{1}{2}$$
 [4 Mks]

c) The 1st term of a geometric progression is 4 and the 6th term is 128. Determine the 8th and 11th terms. [4 Mks]

d) Expand $(x+2)^5$ using binomial theorem expansion [3 Mks]

e) Identify the degree, leading term, and leading coefficient of the polynomial. [3 Mks]

$$4x^2 - x^6 + 2x - 6$$

f) Write the following expressions in the form $a+bi$. [3 Mks]

$$(-3+2i)(4-4i)$$

g) Find the derivative of the following function [3 Mks]

$$y = \frac{1}{x} + \sqrt{x^3}$$

h) Evaluate the following integral

[3 Mks]

$$\int \left(\frac{1}{x^2} + x \right) dx$$

SECTION B: ANSWER ANY TWO QUESTIONS IN THIS SECTION

QUESTION TWO (20 MARKS)

a) Define the term mathematical statement as used in logic and true statements

[2 Mks]

b) Evaluate the following expressions.

[5 Mks]

$$\frac{1 + \sqrt{25} + 3 \times 2 - 8 \div 2}{3 \times 4 - \sqrt{(3^2 + 4^2)} + 1} - \frac{(4 \times 2 + 7 \times 2) \div 11}{\sqrt{9} + 12 \div 2 - 2^3}$$

c) A man gets employment of \$ 750 per month with an annual increment of \$50. What does he earn in 10th year?

[3 Mks]

d) Find the derivative of $y = (x^2 + 1)(x^3 + 3)$

[5 Mks]

e) Consider the following universal set U defined as $U = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$. Consider other three sets defined as

$$A = \{1, 3, 4, 5\}$$

$$B = \{4, 5, 6, 2\}$$

$$C = \{1, 5, 7, 10\}$$

Find the following:

[5 Mks]

(i) $(A \cap B)^c$

(ii) $A \cap (B \cup C)$

QUESTION THREE (20 MARKS)

a) Differentiate between tautology and contradiction

[4 Mks]

b) Express $Z = \left(\frac{3-i}{3+2i} + \frac{2+i}{3-2i} \right)$ in the standard form $x + yi$

[5 Mks]

- c) Find the derivative of $y = (3x^2 + 2)^4$ [4 Mks]
- d) In a group of 70 people, 37 like coffee, 52 like tea and each person likes at least one of the two drinks. How many people like both coffee and tea? [3 Mks]
- e) Write the truth table for the exclusive disjunction of P and Q [4 Mks]

QUESTION FOUR (20 MARKS)

- a) Given the following series 3, 9, 27, . . . what is the n^{th} of the series which gives a value of 59049? [3 Mks]
- b) Solve the simultaneous equations by substitution method [3 Mks]
- $$12x - 3y = 6$$
- $$-7x + 3y = -1$$
- c) In a survey of 600 students in a school, 150 students were found to be taking tea and 225 taking coffee, 100 were taking both tea and coffee. Find how many students were taking neither tea nor coffee? [3 Mks]
- d) Let $f, g : \mathbb{R} \rightarrow \mathbb{R}$ is defined by $f(x) = 2x - 3$ and $g(x) = \frac{x+1}{5}$ respectively. Find f^{-1} and g^{-1} [2 Mks]
- e) Given that $f(x) = 2x + 1$ and $g(x) = x^2 + 2x - 3$ Find: [3 Mks]
- $(f \circ g)(x)$
 - $g(-6)$
- f) Let P be the proposition “Today is Saturday” and Q be “I will go to Nairobi”. Write the following propositions symbolically:
- Today is Saturday or I will go to Nairobi but not both [1 Mk]
 - If and only if today is not Saturday, then I will go to Nairobi. [1 Mk]
- g) Construct the truth table for $\neg (P \vee Q)$ [4 Mks]

QUESTION FIVE (20 MARKS)

- a) State any three logical operators/ connectors and write there symbols as discuss in logic [3 Mks]
- b) Consider the following propositions:
P: Mathematician are generous.
Q: Spiders hate algebra.
Write the compound propositions symbolized by [4 Mks]
i. $\sim P \Rightarrow \sim Q$
ii. $\sim P \Leftrightarrow \sim Q$
- c) Given that $z = 3 + 4i$ and $w = 12 + 5i$, write down the moduli and arguments of [4 Mks]
i) z
ii) zw
- d) Let $S = \{a, b, c\}$. Find all the subsets of S. [4 Mks]
- e) Determine whether or not each of the following relations is a function [3 Mks]
i) $y = 3x + 2$
ii) $y < x + 4$
iii) $y = 2x^2 + 1$
- f) For the following function, determine whether it is one-to-one or onto $f : \mathbb{Z} \rightarrow \mathbb{Z}$ defined by $f(x) = 2x + 1$ [2 Mks]